※注意：請於試卷上「非選擇題作答區」標明題號並依序作答。

※ 禁止使用計算機

Instructions:

- Work need not be shown, only answers will be graded.
- 4 points for each blank and 100 points total.
- Each answer need to be clearly labeled on the answer sheet.
- Use of any device with computer algebra system during the exam will result in zero points.

1. Compute the following limits
   - $\lim_{x\to 0}\left(\cos x+\frac{1}{2}x^2\right)^{\frac{1}{x^4}} = \underline{\quad(1)\quad}$.
   - $\lim_{x\to 0}\frac{3x-\sin 3x}{5x-\tan 5x} = \underline{\quad(2)\quad}$.
2. The graph of $f(x)=(x+1)^{2/3}(x-2)^{1/3}$ has an inflection point at $x = \underline{\quad(3)\quad}$.
3. If $x^5+y^5=33$, then $\left.\frac{d^2y}{dx^2}\right|_{x=1} = \underline{\quad(4)\quad}$.
4. If $\int_1^{2x+1}\frac{f(t)}{e^t}\,dt=\tan^{-1}x$, then $f(3) = \underline{\quad(5)\quad}$.
5. Calculate the following integrals:
   - $\int_0^2\frac{x^3}{(x^2+4)^3}\,dx = \underline{\quad(6)\quad}$.
   - $\int_0^{\ln 2}\sqrt{e^x-1}\,dx = \underline{\quad(7)\quad}$. (Hint: Use $u=\sqrt{e^x-1}$.)
   - $\int_1^3\frac{x-2}{\sqrt{x^2-1}}\,dx = \underline{\quad(8)\quad}$.
6. The integral $\int_0^\infty\frac{(\tan^{-1}x)^4}{x^a}dx$ converges if and only if a is in the interval $\underline{\quad(9)\quad}$.
7. Suppose that $y'=xy-x$ with $y(0)=2$, then $y(2) = \underline{\quad(10)\quad}$.
8. Suppose that $y'+y=2\cos x$ with $y(0)=2$, then $y\left(\frac{\pi}{6}\right) = \underline{\quad(11)\quad}$.
9. Let R be the region below the curve $y=\sin^2 x$ when $0\le x\le\pi$ and V be the volume of the solid obtained by rotating R about the y-axis. Then $V = \underline{\quad(12)\quad}$.

見背面

10. Find the sum

- $\displaystyle\sum_{n=2}^{\infty}\frac{(n-2)!+2^n}{n!} =$ ___(13)___.
- $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2n-1}\left(\frac{1}{\sqrt{3}}\right)^{2n} =$ ___(14)___.

11. The 3th nonzero term in the Maclaurin series of $\ln(2x^3+5)$ is ___(15)___.

12. The 3rd nonzero term of the Mclaurin series of the function

$$f(x)=\begin{cases}\csc x-\cot x, & x\neq 0,\\ 0, & x=0,\end{cases}$$

is ___(16)___.

13. Let $\vec{r}(t)=(e^t\cos t, e^t\sin t, e^t)$, $-1\le t\le 1$.

- The length of the curve is ___(17)___.
- The curvature at the point $t=0$ is ___(18)___.

14. The shortest distance from the origin to the paraboloid $z=\dfrac{x^2+2y^2-36}{4}$ is ___(19)___.

15. $\displaystyle\int_0^1\int_0^2\int_{y/2}^1 yz\cos(x^3-1)\,dx\,dy\,dz =$ ___(20)___.

16. The volume of the solid described by $x^2+y^2\le 1$ and $x^2+y^2+z^2\le 4$ is ___(21)___.

17. The area of the surface $x^2+y^2+z^2=4$, $(x-1)^2+y^2\le 1$, is ___(22)___.

18. Let $\vec{F}(x,y,z)=(\sin y, x\cos y+\cos z, -y\sin z)$, $C:\vec{r}(t)=(\sin t,\cos t,2t)$, $0\le t\le\pi$.
$\displaystyle\int_C \vec{F}\cdot d\vec{r} =$ ___(23)___.

19. Let C be the curve consisting of line segments from (0,0) to (2,1) to (1,2) to (0,0).
$\displaystyle\int_C (x-y)\,dx+(2x+y)\,dy =$ ___(24)___.

20. The flux of the vector field $\vec{F}=(\sin y+x^3, 3yz^2+e^z, 3zy^2)$ through the surface $x^2+y^2+z^2=1$, $z\ge 0$, where the surface is equipped with the upward normal, is ___(25)___.

試題隨卷繳回