※禁止使用計算機

全卷100分，每個答案5分。所有題目均須作答於答案卷上。
不得使用計算機或其他電子計算工具。
The total score of this exam is 100 points, 5 points for each answer.
ALL QUESTIONS have to be answered on your ANSWER SHEET.
The use of digital calculators or other calculating devices are NOT allowed.

Part I 選擇題 Multiple Choice

從(A)、(B)、(C)及(D)選項中選出最適答案，並填寫在答案卷的「選擇題作答區」內。Choose the most suitable answer among the choices (A), (B), (C) and (D) and put it into the "Multiple Choice Answer" section of your ANSWER SHEET.

1. Let f be a smooth function on $\mathbb{R}$.

   (a) If $f'' > 0$ on $\mathbb{R}$, then f' must be increasing on $\mathbb{R}$.
   (b) If $f'' > 0$ on $\mathbb{R}$, then f must be concave upward on $\mathbb{R}$.
   (c) If $f''(a) = 0$, then the point $(a, f(a))$ must be an inflection point.

   Among the above three statements, how many of them are true?
   A) Only one. B) Only two. C) All of them. D) None of them.

2. A function f on $\mathbb{R}$ is *odd* if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$ while it is *even* if $f(-x) = f(x)$ for all $x \in \mathbb{R}$. Suppose that g and h are two functions on $\mathbb{R}$.

   (a) If g is an even function, then the composite function $h \circ g$ must be even.
   (b) If g is an odd function and $\lim_{x\to 0^+} g(x) = 3$, then the limit $\lim_{x\to 0^-} g(x)$ must be -3.
   (c) If g is an even function and $\lim_{x\to 0^+} g(x) = 3$, then the limit $\lim_{x\to 0} g(x)$ must be 3.

   Among the above three statements, how many of them are true?
   A) Only one. B) Only two. C) All of them. D) None of them.

3. Consider the integral $I_{p,r} := \iint\limits_D \frac{1}{(x^2+y^2)^{p/2}}\, dA$, where D is the region bounded by two concentric circles centred at the origin with radii r and 1 respectively, $0 < r < 1$. Let $J_p := \lim_{r\to 0^+} I_{p,r}$.

   (a) J_p is convergent if $p < 1$.
   (b) J_p is convergent if $1 < p < 2$.
   (c) J_p is convergent if $2 < p$.

   Among the above three statements, how many of them are true?
   A) Only one. B) Only two. C) All of them. D) None of them.

4. Let $f(x,y) = \begin{cases} \dfrac{x\sin y^2}{1-e^{x^2+y^4}} & \text{if } (x,y) \neq (0,0)\,, \\ 0 & \text{if } (x,y) = (0,0)\,. \end{cases}$

   (a) $\lim_{x\to 0}\lim_{y\to 0} f(x,y) = 0$.
   (b) $\lim_{x\to 0}\lim_{y\to 0} f(x,y) = \lim_{y\to 0}\lim_{x\to 0} f(x,y)$.
   (c) $\lim_{(x,y)\to(0,0)} f(x,y) = 0$.

   Among the above three statements, how many of them are true?
   A) Only one. B) Only two. C) All of them. D) None of them.

見背面

5. (a) The series $\sum_{n=1}^{\infty} n\cos(n\pi)\sin\frac{1}{n}$ is absolutely convergent.
   (b) The series $\sum_{n=1}^{\infty} \sin(n)\,\frac{n^2+1}{5^n}$ is absolutely convergent.
   (c) The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^{\sqrt{2}-1}}$ is absolutely convergent.

   Among the above three statements, how many of them are true?

   A) Only one. B) Only two. C) All of them. D) None of them.

Part II 填充題 Fill in the blanks

找出最適合填入下面空格的答案，並把空格上的題號和你的答案清楚填寫於答案卷上。請按題號的順序填寫答案，並無須填寫答案推導過程。Find a suitable answer to fill in each of the blanks below. Write the LABEL ON THE BLANK as well as YOUR ANSWER clearly in your ANSWER SHEET. Please write your answers in the order of the numbers of the labels. Explanation to your answer is NOT needed.

6. $\lim_{n\to\infty} \frac{(1^2+2^2+\cdots+n^2)}{(\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n})^2} =$ ___(6)___.

7. Let $f(x) = \int_0^{\frac{\pi}{2}+2\sin x} \sin(y + x\cos y)\, dy$. It follows that $f'(0) =$ ___(7)___.

8. Let $f(x) = \arctan(x^2) + \frac{x^2-1}{x^2+1}$. The graph of f has a horizontal asymptote represented by the equation ___(8)___ and the global minimum value of f is ___(9)___.

9. $\int_0^1 \frac{1}{(x^2+1)^2}\, dx =$ ___(10)___.

10. The third term of the Maclaurin series (i.e. the Taylor series centred at $x = 0$) of $\arcsin(3x)$ is ___(11)___ (note that the answer should be a monomial in x; the term of x^0 is counted as the 0-th term). The radius of convergence of the series is ___(12)___.

11. If $y = y(x)$ satisfies the differential equation $x^2y' + 3xy = 2\ln x$ for $x > 0$ with $y(1) = 2$, then $y(x) =$ ___(13)___.

12. Let C be a variable path in the xy-plane of arc-length 1 starting at the point $(\sqrt{3}, 1)$ and ending at the point (a, b). Suppose that $G(a,b) := \int_C \nabla f \cdot dr$, where $f(x,y) := \arctan\frac{y}{x}$, is a function in a and b. Then, G attains its maximum at $a =$ ___(14)___ and $b =$ ___(15)___, and the maximum value of G is ___(16)___.

13. If a and b are positive constants and if $\max\{p,q\}$ denotes the maximum between the numbers p and q, the iterated integral $\int_0^a \int_0^b e^{\max\{b^2x^2,\, a^2y^2\}}\, dy\, dx =$ ___(17)___.

14. Let E be a tetrahedron（四面體）in $\mathbb{R}^3$ bounded by the planes $x+y+z=3$, $x=2z$, $y=0$ and $z=0$. Let also $\mathbf{F} := (x-y)\,\mathbf{i} + (y^2+z^2)\,\mathbf{j} + e^{x^3}\,\mathbf{k}$.
    (a) $\operatorname{curl}\mathbf{F} =$ ___(18)___.
    (b) If S_1 is the boundary surface of E (including all faces) endowed with the outward orientation, one has $\iint_{S_1} \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} =$ ___(19)___.
    (c) If S_2 is the surface obtained from S_1 by removing the face in the xy-plane while keeping the orientation from S_1 on all other faces, one then has $\iint_{S_2} \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} =$ ___(20)___.

試題隨卷繳回