※ 注意：請於答案卷上依序作答，並應註明作答之部份及其題號。

(1) (20 pts) Find a Jordan form J of the upper triangular matrix

$$\begin{pmatrix} 2 & 4 & -8 \\ 0 & 0 & 4 \\ 0 & -1 & 4 \end{pmatrix}$$

and find a matrix T such that $T^{-1}AT = J$.

(2) (20 pts) Let I_n be the $n \times n$ identity matrix, v be a $n \times 1$ vector and $A = I_n + vv^T$.
(i) Show that A is invertible and $det(A) = 1 + v^T v$ where v^T is the transpose of v.
(ii) Find an explicit formula of A^{-1}.

(3) (20 pts) Let A be an $p \times q$ matrix of rank α and B a $r \times s$ matrix of rank β. Let $M = \{C| \quad C \text{ is a } q \times r \text{ matrix such that } ACB = 0\}$
(i) Prove that M is a vector space.
(ii) Find the dimension of the vector space M.

(4) (20 pts) Let V be the vector space of 3×3 real matrices that are skew symmetric, i.e. $A^T = -A$ (where A^T is the transpose of A. Prove the expression

$$\langle A, B \rangle = \frac{1}{2} Tr(AB^T)$$

defines a inner product on V, and exhibit an orthonormal basis of V with respect to this inner product. Here $Tr(AB^T)$ is the trace of AB^T.

(5) (20 pts) Let V be a vector space over $\mathbb{C}$ of dimension n, and let $T : V \mapsto V$ be an invertible linear map such that $T^{-1} = T$.
(i) Prove that T is diagonalizable.
(ii) Denote by S the vector space of linear transformations from V to V that commute with T. Find a formula for $dim(S)$ in terms of n and the trace of T.

試題隨卷繳回