

※注意：禁止使用計算機

※注意：請於試卷上「非選擇題作答區」標明題號並依序作答。

- 考試不可使用計算機。
- 請於試卷上「非選擇題作答區」依序作答。
- 保留答案簿前兩面(四頁)為答案區，第五頁之後為計算草稿區。
- 第1至6題為填充題，答案請寫在答案簿第一面(兩頁)，任何計算皆不計分。
- 第7題為計算題，請將計算過程詳細寫在翻面的第三、四頁，只寫答案不計分。

1. Assume a (real valued) function  $y = f(x)$  satisfy  $y^5 + 9y = x^3 + x$ . Suppose  $g(x)$  is a (real valued) function satisfying  $f(g(x)) = x$ .
  - (a) (5%) The value  $g(1)$  must be 2. True or false? Answer: (1).
  - (b) (10%) The derivative  $g'(1) =$  (2).
2. An experiment detects that a particle at  $(1, 0)$  (on the  $xy$ -plane) is moving towards the north at the speed 3 meters per minute. At the same time another particle at  $(-2, 1)$  is moving towards the east at the speed 4 meters per minute.
  - (a) (5%) Let  $s(t)$  be the distance (in meters) between the two particles in  $t$  minutes.  $s(t) =$  (3).
  - (b) (10%) Suppose at  $t = t_0$  the two particles are closest to each other. Then  $t_0 =$  (4).
3. (10%) Compute the indefinite integral  $\int \frac{dx}{(x^2-1)(x+1)} =$  (5).
4. (10%) Let  $\Omega$  be a region (on the  $xy$ -plane) enclosed by  $x = \sqrt{\ln y}$ ,  $x = 0$  and  $y = e$ . Let  $S$  be the solid obtained by revolving  $\Omega$  about the  $y$ -axis. Let  $V$  be the volume of  $S$ .  $V =$  (6).
5. Let  $f(x, y) = x^2 - e^{xy^2}$  and the surface  $S$  be the graph of the function  $z = f(x, y)$ . Let  $P = (1, 0, 0) \in S$  and  $p = (1, 0)$  in the  $xy$ -plane.
  - (a) (10%) If the unit vector  $u$  (in the  $xy$ -plane) at  $p$  is the direction (among all directions at  $p$ ) along which the height (i.e. the value of  $z$ ) of  $S$  increases most rapidly, then  $u =$  (7).
  - (b) (10%) Write  $H$  for the plane  $x + 2y + 3z = 1$  and the curve  $C$  for the intersection  $S \cap H$ . Let  $L$  be the tangent line to  $C$  at  $P$  and  $N$  be the plane perpendicular to  $L$  at  $P$ . Then the equation of  $N$  is (8).
6. Let  $R$  be the region enclosed by  $y = x$ ,  $y = x - 2$ ,  $y = 1$  and  $y = 0$ .
  - (a) (10%) Let  $A$  be the area of  $R$ .  $A =$  (9).
  - (b) (5%) The double integral  $\iint_R \sqrt{x-y} \, dx dy =$  (10).
7. (15%) Let  $\Omega$  be the region  $\{(x, y) | (x-1)^2 + y^2 < 1\}$ . Evaluate the double integral  $\iint_{\Omega} \frac{1}{x} \, dx dy$ .

試題隨卷繳回