

要有演算過程或寫出理由，否則扣分。

1. (10%) Find the value of  $c$  such that the limit exists and evaluate the limit.

$$\lim_{x \rightarrow 1} \left( \frac{1}{x-1} - \frac{c}{x^3-1} \right)$$

2. (15%) Find the extreme values of  $f(x) = x^x$  for  $x > 0$ .
3. (15%) Evaluate the indefinite integral.

$$\int \frac{xdx}{\sqrt{1+x^2} + \sqrt{(1+x^2)^3}}$$

4. (15%) Find an equation for the line through the origin (原點) tangent to the graph of  $y = \ln x$ .
5. (15%) Find the sum of the series or show that it diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

6. (a) (5%) Give the definition of the directional derivative of  $f(x, y)$  at  $(a, b)$  in the direction of a unit vector  $\mathbf{u} = (u, v)$ ,  $u^2 + v^2 = 1$ .
- (b) (5%) Show that the directional derivative of the following function exists in every direction  $\mathbf{u} = (u, v)$  at  $(0, 0)$ , where

$$f(x, y) = \begin{cases} \frac{xy^2}{x^2+y^4}, & \text{if } (x, y) \neq (0, 0); \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

- (c) (5%) Show that the above function  $f(x, y)$  is not differentiable at  $(0, 0)$
7. (15%) Use the transformation  $x = u/v$ ,  $y = v$  to evaluate the double integral

$$\iint_R xy dA,$$

where  $R$  is the region in the first quadrant (象限) bounded by the lines  $y = x$ ,  $y = 3x$ , and the hyperbolas  $xy = 1$ ,  $xy = 3$ .