(1) (16 %) Let $f(x)$ and $g(x)$ be two differentiable functions such that

$$\frac{d}{dx}f(x) = -g(x),$$

$$\frac{d}{dx}xg(x) = xf(x).$$

(a) Show that between two consecutive roots of $f(x) = 0$, $g(x) = 0$ has a root.
(b) Show that between two consecutive roots of $g(x) = 0$, $f(x) = 0$ has a root.

(2) (16 %) A man can swim at a m/sec and run at b m/sec. If he stands at point $A = (50, 0)$ on the edge of a circular swimming pool of radius 50 m with its center at the origin, find his optimum path from A to $B = (0, 50)$.

(3) (17 %) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence defined by $a_1 = 1, a_{n+1} = 1 + \frac{1}{1+a_n}, n \geq 1$.
(a) Show that $\lim_{n\to\infty} a_n$ exists.
(b) Find $\lim_{n\to\infty} a_n$.

(4) (17 %) Let S be the surface $z = 4 - x^2 - y^2$. The temperature function in space S is $T(x, y, z) = x^2y + y^2z + 2x + 4y + z$. Let $P = (1, 1, 2)$ be a point in S. Among all the possible directions tangential to S at P, which direction will make the rate of change of temperature at P a maximal?

(5) (17 %) Find the volume of the solid bounded by the surface $(x^2 + y^2 + z^2)^2 = a^2(x^2 + y^2 - z^2)$, $a > 0$.

(6) (17 %) Let C be the curve $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}, 0 \leq t \leq 2\pi$. Let S be the surface obtained by connecting any point $(x, y, z) \in C$ to $(0, 0, z)$ with a line segment. Find the area of S.

試題隨卷繳回