

※ 請在答案卷上標明題號依序作答

一. Let $\int x^n(1-x)^2 dx = F(x) + c$ and $\int x^2(1-x)^n dx = G(x) + c$ where n is a positive integer. (15%)

(1) Find the relation between $F(x)$ and $G(x)$.

(2) Find the integral $\int_0^1 x^2(1-x)^{21} dx$.

二. Let C be the curve formed by the intersection of the cylinder $x^2 + y^2 = 5$ and the plane $x - 2y - 5z = 0$. Find the minimum distance from the point $(0, 0, 2)$ to the curve C . (20%)

三. Let $I_n = \int_1^e t(\ln t)^n dt$. (15%)

(1) Find the relation between I_n and I_{n-1} .

(2) Find the values I_1, I_2, I_3, I_4 .

四. Let r be the radius of convergence of the power series $\sum_{n=1}^{+\infty} n x^{3n}$. (15%)

(1) Find r .

(2) Let $g(x) = \sum_{n=1}^{+\infty} n x^{3n}$ for any $|x| < r$.

Find $g(x), g(0.5)$.

五. Find the integral $\int_0^{\ln 2} \left(\int_{e^y}^2 \frac{3}{\ln x} dx \right) dy$ (15%)

六. Let $\vec{F}(x, y, z) = (2x + 3xy^2)\vec{i} + (xy^2 + 3yx^2)\vec{j} + (z^3 - 2xyz)\vec{k}$ be a vector field and $S: x^2 + y^2 + z^2 = 49$ be a sphere. Find the flux $\iint_S \vec{F} \cdot \vec{n} dS$, where \vec{n} is the outward unit normal vector of S . (20%)

試題必須隨卷繳回