Instructions

(1) Question 1-9 are multiple-answer questions (10 points for each question). If there's no further instruction, please choose argument(s) which is(are) true. The number of correct answers for each question could be from 1 to 5. For each wrong answer in a question, you are deducted 4 points until you get 0 point in that question. For instance, suppose the correct answers are abc, but your answers are acd, then you get a right, miss b, get c right, wrongly include d, correctly miss e. Thus, for this question, there are two wrong answers (miss b and wrongly include d), hence you get 10-4-4=2 points. If you leave a question blank (don't choose any answer), that would be viewed as not answering the question, and you get 0 point. Please answer question 1-9 in the answer card.

(2) Question 10 is a calculation question (10 points). Please write your answer in the answer sheet.

1. Suppose you could vary prices and income (p_x, p_y, I) at any positive level and you could observe individuals' corresponding consumption choices (x^*, y^*). In each following option, you need to distinguish between two preferences. Please choose the option(s) in which two preferences are distinguishable according to the observational data mentioned above.

   Notes on preference:

   Weak preference relation $\succsim$:

   $(x_1, y_1) \succsim (x_2, y_2)$ means "the consumer wants (x_1, y_1) at least as much as (x_2, y_2)" or "the consumer weakly prefers (x_1, y_1) to (x_2, y_2)"

   Indifference preference relation $\sim$:

   $(x_1, y_1) \sim (x_2, y_2) \leftrightarrow (x_1, y_1) \succsim (x_2, y_2)$ & $(x_2, y_2) \succsim (x_1, y_1)$, which reads "the consumer is indifferent between (x_1, y_1) and (x_2, y_2)"

   Strong preference relation $\succ$:

   $(x_1, y_1) \succ (x_2, y_2) \leftrightarrow (x_1, y_1) \succsim (x_2, y_2)$ & $(x_2, y_2) \not\succsim (x_1, y_1)$ where $\not\succsim$ means not $\succsim$.

   Notes on utility function:

   A utility function $u(\cdot)$ can represent/characterize preference means:

   $(x_1, y_1) \succsim (x_2, y_2) \leftrightarrow u(x_1, y_1) \geq u(x_2, y_2)$

   $(x_1, y_1) \sim (x_2, y_2) \leftrightarrow u(x_1, y_1) = u(x_2, y_2)$

   $(x_1, y_1) \succ (x_2, y_2) \leftrightarrow u(x_1, y_1) > u(x_2, y_2)$

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(a) Preference A1 is described by a utility function: $u_{A1}(x,y) = x^{\alpha} \cdot y^{\beta}$
Preference A2 is described by a utility function:

$$u_{A2}(x,y) = \begin{cases} x^{\alpha} \cdot y^{\beta} + 7, \text{ if } x = y \\ x^{\alpha} \cdot y^{\beta}, \text{ if otherwise} \end{cases} \quad (1)$$

$$\alpha, \beta > 0$$

(b) Preference B1 is described by a utility function: $u_{B1}(x,y) = x^{\alpha} \cdot y^{\beta}$
Preference B2 is described as:

$$\begin{cases} (x_1,y_1) \succ_{B2} (x_2,y_2), \text{ if } {x_1}^{\alpha} \cdot {y_1}^{\beta} > {x_2}^{\alpha} \cdot {y_2}^{\beta} \\ (x_1,y_1) \succ_{B2} (x_2,y_2), \text{ if } {x_1}^{\alpha} \cdot {y_1}^{\beta} = {x_2}^{\alpha} \cdot {y_2}^{\beta} \;\&\; x_1 = y_1 \;\&\; x_2 \neq y_2 \\ (x_1,y_1) \sim_{B2} (x_2,y_2), \text{ if } {x_1}^{\alpha} \cdot {y_1}^{\beta} = {x_2}^{\alpha} \cdot {y_2}^{\beta} \;\&\; x_1 \neq y_1 \;\&\; x_2 \neq y_2 \end{cases} \quad (2)$$

$$\alpha, \beta > 0$$

(c) Preference C1 is described by a utility function: $u_{C1}(x,y) = x^{\alpha} + y^{\alpha}$
Preference C2 is described by a utility function: $u_{C2}(x,y) = x^{\beta} + y^{\beta}$

$$1 > \alpha > \beta > 0$$

(d) Preference D1 is described by a utility function: $u_{D1}(x,y) = x^{\alpha} + y^{\alpha}$
Preference D2 is described by a utility function: $u_{D2}(x,y) = x^{\beta} + y^{\beta}$

$$\alpha > \beta > 1$$

(e) Preference E1 is described by a utility function: $u_{E1}(x,y) = (x-3)^2 + (y-7)^2$
Preference E2 is described by a utility function: $u_{E2}(x,y) = \frac{-1}{(x-3)^2+(y-7)^2}$

2. Consider N potential firms could produce the same good ($i = 1, 2, 3, ...N$; N is an extremely large positive integer). Each firm uses labor and capital to produce the good with the same production function:

$$q_i = f(L_i, K_i) = \min\{{L_i}^{\frac{1}{2}}, {K_i}^{\frac{1}{2}}\}$$

In the labor market and capital market, all firms are price takers. The price of labor is $w > 0$, and the price of capital is $r > 0$. However, the fixed cost varies across firms. Firm i's fixed cost is:

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$$F_i = i^2$$

Finally, we assume that the market of the output is competitive without restrictions on entry and exit. Please consider only the case in the long-run.

(a) If w stays the same but r increases, firms will use relatively more labor in production ($\frac{L_i^*}{K_i^*}$ is higher).

(b) Every firm has the same marginal cost function which is linear.

(c) Because of competition, firms will earn 0 profit.

(d) Suppose $w = r = \frac{1}{2}$. If the price of the output is larger than 4 and less than 6, there will be 2 firms staying in the market.

(e) Suppose $w = r = \frac{1}{2}$. If in the equilibrium, aggregate quantity of the good is 9, there are 3 firms staying in the market.

3. Suppose there are N smokers and N non-smokers in a room (N is a positive integer), and each of them has m dollars. A smoker i decides his consumption of cigarettes (s_i can be non-negative real numbers). We denote $p > 0$ as the price of one cigarette. Smoker i only cares about his consumption of cigarettes and his wealth left. His utility would be:

$$U_i = u(s_i) + m - p \cdot s_i$$
$$u' > 0, u'' < 0, \lim_{s_i \to 0} u'(s_i) = \infty$$

A non-smoker j never consumes cigarettes, and he hates the second-hand smoke in the room. His utility would be:

$$V_j = m - h(\Sigma_{i=1}^{N} s_i)$$
$$h' \geq 0, h'' > 0, \lim_{S \to 0} h'(S) = 0$$

(a) If every individual is selfish and there's no transfer or bargaining, the equilibrium would be efficient when $N = 1$.

(b) For some $N \geq 2$, it's possible to find efficient outcome in which $s_i^* \neq s_{i'}^*$, $U_i > u(s_i^*)$, and $U_{i'} > u(s_{i'}^*)$ for some $i \neq i'$

(c) Given symmetric efficient outcome ($s_i^* = s^*, U_i = U^*, \forall i; V_j = V^*, \forall j$), the individual-level consumption of cigarettes (s^*) should decrease as N increases. (Please only consider interior solution.)

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(d) Given symmetric efficient outcome ($s_i^* = s^*, U_i = U^*, \forall i; V_j = V^*, \forall j$), the aggregate consumption of cigarettes ($N{\cdot}s^*$) should decrease as N increases. (Please only consider interior solution.)

(e) Given symmetric efficient outcome ($s_i^* = s^*, U_i = U^*, \forall i; V_j = V^*, \forall j$). If N is large enough (still finite), it's possible to find $s^* = 0$. (You need to consider the possibility of corner solution)

4. In this question, consider an exchange economy with two individuals ($i = 1, 2$) and two goods (x, y). Suppose (x_i, y_i) is individual i's consumption and (x_{i0}, y_{i0}) is individual i's initial endowment. Individuals' utility functions are:

$$u_1(x_1, y_1) = {x_1}^{\alpha}{\cdot}{y_1}^{\beta}, \alpha > 0, \beta > 0$$
$$u_2(x_2, y_2) = x_2$$

Initial endowment follows:

$$x_{10} + x_{20} = a > 0, y_{10} + y_{20} = b > 0$$
$$x_{i0} \geq 0, y_{i0} \geq 0, \forall i$$

(a) Contract curve is a smooth curve.

(b) It's possible to have $(p_x, p_y) = (p, 0)$ with $p > 0$ as general equilibrium (Walrasian equilibrium).

(c) It's possible to have $(p_x, p_y) = (0, p)$ with $p > 0$ as general equilibrium (Walrasian equilibrium).

(d) The result of the First Welfare Theorem always holds in this economy.

(e) The result of the Second Welfare Theorem always holds in this economy.

5. Consider a parent with a child. The probability of the child being sick is $\frac{1}{2}$. Given that the child is sick, the probability that the child passes away is also $\frac{1}{2}$. Suppose that the parent has $w_0 > 0$ as initial wealth. If the child gets sick, the parent will definitely spend $c > 0, c < w_0$ saving the child because the parent cares about the child. The parent's utility function is $ln(w)$ when the child is alive, and the parent's utility function is $\alpha{\cdot}ln(w), 0 < \alpha < 1$ ($ln(\cdot)$ means natural log). $\alpha < 1$ means that losing the child hurts the parent. The parent is facing either one of the following two cases of insurance provision.

Case 1-insurance on child's being sick:

The parent can purchase $q_s \geq 0$ units of the insurance. When the child gets sick, the insurance company will pay q_s dollars to the parent. The unit price of insurance is p_s which makes the insurance company earn 0 profit in expectation.

Case 2-insurance on child's death:

The parent can purchase $q_d \geq 0$ units of the insurance. When the child passes away, the insurance company will pay q_d dollars to the parent. The unit price of insurance is p_d which makes the insurance company earn 0 profit in expectation.

(a) p_s is $\frac{1}{2}$ and p_d is $\frac{1}{4}$

(b) If losing the child hurts the parent more (smaller α), the parent will purchase more units of insurance under both cases.

(c) Under Case 1, the parent will purchase some positive units of insurance to cover the medical cost (c).

(d) Under Case 2, the parent will purchase some positive units of insurance to cover the medical cost (c).

(e) The parent will purchase more units of insurance under Case 2 than under Case 1.

6. There are two types of consumers for a monopolist's product: enthusiasts and normies. The product is indivisible and can only be sold in integer units. The population consists of equal number of enthusiasts and normies. Table 1 shows their respective demand schedules and the combined demand schedule. There is no fixed cost of production. The monopolist faces a constant marginal cost of \$1.5.

(a) Suppose the monopolist cannot tell enthusiasts from normies. So the monopolist can only charge the same price to both types of consumers. The profit maximizing price is \$5.

(b) Suppose the monopolist cannot tell enthusiasts from normies. So the monopolist can only charge the same price to both types of consumers. Marginal revenue from the third unit, that is, the change in the monopolist's total revenue when he increases quantity from 2 to 3, is \$2.5.

(c) The outcome where the monopolist produces 9 units, the enthusiasts consumes 6 units and the normies consume 3 units is Pareto efficient.

price	enthusiasts' quantity demanded	normies' quantity demanded	total quantity demanded
7	1	0	1
6	2	0	2
5	3	0	3
4	4	1	5
3	5	2	7
2	6	3	9
1	7	4	11
0	8	5	13

Table 1: Price Schedule for Q??

(d) Suppose the monopolist can distinguish enthusiasts from normies and can charge these two different types of consumers different prices. Total consumer surplus is strictly lower when the monopolist can price discriminate and charge different prices than when the monopolist can only charge a uniform price.

(e) Suppose the monopolist can identify enthusiasts and normies and charge these two different types of consumers different prices. Suppose the monopolist changed the product design that doesn't affect the normies' demand cruve but shifts enthusiasts' demand curve upward vertically by \$1.5. Under the new product design, the monopolist faces an increasing marginal cost curve

$$MC(Q) = \begin{cases} 0.3 & \text{if } Q \leq 3 \\ 0.3 + 0.6(Q-3) & \text{if } Q > 3 \end{cases}$$

where Q stands for total quantity produced. Since the new design does not affect the normies' demand curve, the profit maximizing price the monoopolist charges the normies will remain the same.

7. The village of Danan has two coffee shops. Each produces coffee at zero marginal cost. There are two coffee shops in Danan: Ace Coffee and Better Coffee. The two coffee shops simultaneously choose their prices. Prices have to be non-negative. Let p_A denote the price at Ace Coffee and p_B denote the price at Better Coffee. Given price profile (p_A, p_B), the market share of Ace Coffee is

$$D_A(p_A, p_B) = \begin{cases} 0 & \text{if } p_A - p_B > 4 \\ 1 - \frac{p_A - p_B}{4} & \text{if } 0 \leq p_A - p_B \leq 4 \\ 1 & \text{if } p_A - p_B < 0 \end{cases}$$

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and the market share of Better Coffee is

$$D_B(p_A, p_B) = \begin{cases} 1 & \text{if } p_A - p_B > 4 \\ \frac{p_A - p_B}{4} & \text{if } 0 \le p_A - p_B \le 4 \\ 0 & \text{if } p_A - p_B < 0 \end{cases}.$$

(a) The strategies of Ace Coffee and Better Coffee are strategic complements: one shop's best response is increasing in the other shop's strategy.

(b) Because strategies are strategic complements, there are multiple Nash equilibria in this game.

(c) There is a Nash equilibrium in which Ace Coffee charges price of \$2

(d) Suppose Ace Coffee chooses its price first. After observing Ace Coffee's price, Better Coffee then chooses its price. The profit maximizing price for Ace Coffee is then \$4.

(e) Suppose Ace Coffee chooses its price first. After observing Ace Coffee's price, Better Coffee then chooses its price. Better Coffee's profits are lower in this situation than in the situation where both choose prices simultaneously without knowing the other firm's price.

8. The government of Wonderland gives alcohol license to only one firm. Firm A and B are both interested in obtaining the license. Having the alcohol license for is worth \$3 million dollars in total for firm A, and \$2 million dollars in total for firm B. The government official in charge of this decision is known to be corrupt and the firms can influence his decision by offering bribes. If firm A and B offer different amount of bribes, the firm offering a strictly higher amount of bribe will be granted the license. If firm A and B offer equal amount of bribe, then firm A gets the license with probability 1/3 whereas firm B gets the license with probability 2/3. A firm cannot take the bribe back even if it does not obtain the license. Each firm's payoff is equal to the value of the license times the probability of getting the license minus the bribe. For example, if firm A pays bribe of x_A million dollars and obtains the license with probability 1/3, then firm A's payoff is equal to $3 \times \frac{1}{3} - x_A$. Firm A and B approach the official in secret without knowing what the other firm has offered. For $i = A, B$, let x_i denote firm i's pure strategy that gives x_i million in bribery. Assume that x_i has to be a non-negative integer. That is, $x_i = 0, 1, 2, \cdots$.

(a) $x_B \geq 3$ is strictly dominated for firm B.

(b) It is a Nash equilibrium for firm A to offer \$1 million dollars and firm B to offer no bribery.

(c) This game has a pure strategy Nash equilibrium.

(d) There is a Nash equilibrium in which firm B randomizes uniformly between offering nothing, offering \$1 million dollars, and offering \$2 million dollars (so $x_B = 0, 1, 2$ each with probability 1/3).

(e) There is a Nash equilibrium in which firm B randomizes uniformly between offering nothing and offering \$1 million dollar (so $x_B = 0, 1$ each with probability 1/2).

9. Ben has a brilliant business idea and wants to start a company based on that idea. To make a prototype, he needs an initial investment of $I = 100$. However, he has no capital of his own and thus goes to Venture Capitalist (VC) for investment. If VC does not invest, then both VC and Ben get 0. If VC invests I, then Ben will choose whether to work or to shirk. If Ben works hard, he will turn his idea into reality and the value of his startup will become $V = 160$. If Ben shirks and diverts the investment, he can pocket the entirety of the investment $I = 100$ and the value of his startup will becom 0. Suppose VC gets a share $s = 80\%$ of the company if he invests I in the company. Therefore, if VC invests and Ben works, then VC's payoff is $sV - I = 28$ and Ben's payoff is $(1 - s)V = 32$. If VC invests and Ben shirks, then VC's payoff is $-I = -100$ and Ben's payoff is $I = 100$.

(a) In the unique backward induction solution, VC will invest and Ben will work hard, because Ben gets $(1 - s)V = 32$ by working hard after VC invests whereas Ben gets 0 if VC does not invest.

(b) There is no backward induction solutions in which VC invests.

(c) Suppose VC demands only $\frac{5}{8}$ of the company from his investment so that VC just breaks even with the investment, then VC invests in the unique backward induction solution.

(d) Suppose VC demands 80% of the company from his investment. Suppose regulation is much more stringent and Ben can divert only 50% of the investment if Ben shirks. That is, if Ben shirks, then VC's payoff is $-I = -100$ whereas Ben's payoff is $\alpha I = 50$. Then VC invests in the unique backward induction solution.

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(e) Suppose VC demands 80% of the company from his investment and he can get the entirety of the investment if he shirks. Suppose it is commonly known that Ben will have a second business idea and want to do a second startup. Suppose the relevant parameters for his second startup is exactly the same as described up. So, the exact same situation will repeat twice. Then, future concern will discipline Ben. There is a SPNE in this twice-repeated game in which VC invests in Ben's first startup.

10. Quality θ of used cars in the town of Nowhere is distributed according to the probability density function f where $f(\theta) = 2\theta$ for $\theta \in [0,1]$. So, for $0 \le a < b \le 1$, the expected value of θ conditional on $\theta \in [a,b]$ is equal to $\frac{2}{3}\frac{b^2+ab+a^2}{b+a}$. In particular, the expected value of θ conditional on $\theta \in [0,b]$ is equal to $\frac{2}{3}b$. If the owner of a quality θ used car keeps the car, then his payoff is equal to 5θ. Because the used car would be new to people other than the original owner and add spice to life, the payoff of a quality θ used car to a potential buyer is 9θ. So, if the owner of a used car with quality θ sells the car at price p, then his payoff is $p - 5\theta$ while the payoff to the buyer is $9\theta - p$.

Suppose the owner of the used car knows the quality of his car but the buyer does not. In equilibrium, what is the maximum possible quality $\overline{\theta}$ of a car that is offered for sale in the market? If owners of all used car qualities offer their car for sale in some equilibrium, then $\overline{\theta} = 1$. If owners of all used car qualities keep their car in every equilibrium, then $\overline{\theta} = 0$. If owners of cars of some quality levels offer their cars for sale while others don't in some equilibrium, then $\overline{\theta}$ is such that owners of used cars with quality $\theta > \overline{\theta}$ will keep their cars in every equilibrium while owners of used cars with quality $\theta < \overline{\theta}$ will sell their car in some equilibrium.

Just write down your answer of $\overline{\theta}$.