題號： 296
科目：數學(A)
節次： 4

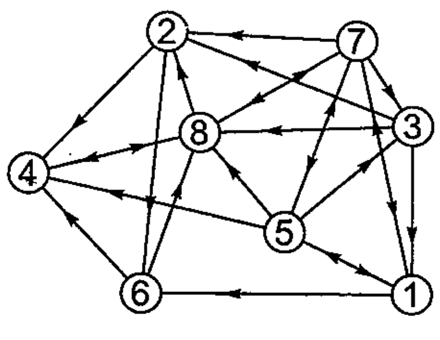
國立臺灣大學 114 學年度碩士班招生考試試題

請先在試卷第一頁繪製以下表格，然後將答案填入。

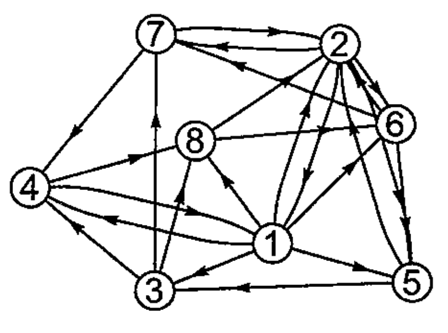
Please draw the following table on the first page of the answer sheets and fill in the answers accordingly.

1		7	
2		8	(A)
3			(B)
4		9	
5		10	
6			

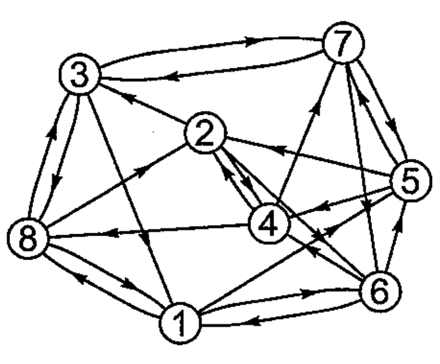
1. (5%) Which ones of the following directed graphs are Eulerian? ________



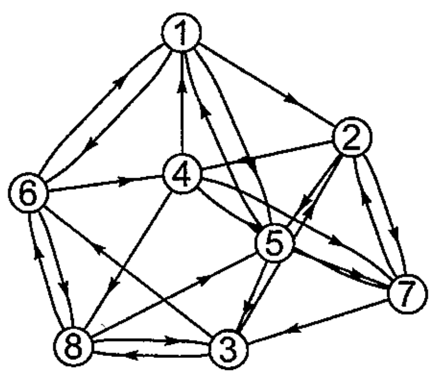
(A)



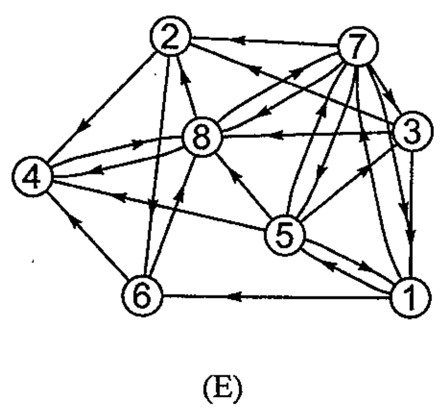
(B)



(C)



(D)



(E)

2. (5%) There are ________ satisfying truth assignments to

$$(\neg w \wedge x \wedge \neg y) \vee (\neg w \wedge \neg x \wedge y \wedge \neg z)$$

見背面

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3. (10%) Let there be N binary relations from $\{A, B, C, D\}$ to $\{1, 2, 3, 4, 5\}$. Calculate $N \bmod 13$: ________

4. (10%) Derive the solution for a_n that satisfies the recurrence equation $a_n = -3a_{n-1} + 10\,a_{n-2}$ with $a_0 = 3$ and $a_1 = 2$: ________

5. (10%) The generating function in partial fraction decomposition for the above recurrence equation is ________

6. (10%) The number of non-negative integer solutions of $x_1 + x_2 + \cdots + x_4 \leq 7$ equals ________

7. (10%) Which ones of the following sets are linearly independent? Points will be counted only if all the answers are correct.
   (A) $\{-x^2 + 3x + 6, x^3 + 2, x^3 - 3x^2 + 5\}$ in $P_3(R)$.
   (B) $\left\{\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\right\}$ in $M_{2\times2}(R)$
   (C) $\left\{\begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}\right\}$ in $M_{3\times2}(R)$
   (D) $\left\{\begin{pmatrix} 2 \\ 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}\right\}$ in R^4
   (E) $\left\{\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}\right\}$ in R^3

8. Let $V = P_3(x)$ be a subspace of $P(x)$ with inner product $<f, g> = \int_{-1}^{1} f(x)g(x)dx$.
   (A) (10%) Find the matrix A with respect to the basis $\{1, x, x^2, x^3\}$ in V such that $<f, g> = [f]^T A[g]$.
   (B) (10%) Find the Fourier coefficient of x^5 along $x^3 - \frac{3}{5}x$.

9. (10%) $A = \begin{bmatrix} 1 & a & a^2 & a^3 & \cdots & a^n \\ a & 1 & a & a^2 & \cdots & a^{n-1} \\ a^2 & a & 1 & a & \cdots & a^{n-2} \\ a^3 & a^2 & a & 1 & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & a \\ a^n & a^{n-1} & a^{n-2} & \cdots & a & 1 \end{bmatrix}$, where $a \in R$ and n is a positive integer. Find the product of all the eigenvalues of A in the simplest form.

10. (10%) Given $A = \begin{bmatrix} 2 & 1 & -2 & 2 & 0 \\ 2 & 3 & -4 & 2 & 5 \\ 1 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 4 & -1 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix}$ and a polynomial $f(x) = x^4 - 7x^3 + 13x^2 - 6x + 1$. Find the largest eigenvalue of $f(A)$.