1. Please indicate the following statements as true or false. (10%)
   (a) If S is a linearly dependent set, then each vector in S is a linear combination of other vectors in S.
   (b) Similar matrices always have the same eigenvalues.
   (c) If the homogeneous system corresponding to a given system of linear equations has a solution, then the given system has a solution.
   (d) Two distinct eigenvectors corresponding to the same eigenvalue are always linearly dependent.
   (e) An inner product is linear in both components.

2. At the end of May, a furniture store had the following inventory.

	Early American	Spanish	Mediterranean	Danish
Living room suites	4	2	1	3
Bedroom suites	5	1	1	4
Dining room suites	3	1	2	6

Record these data as a 3 x 4 matrix M. To prepare for its June sale. the store decided to double its inventory on each of the items listed in the preceding table. Assuming that none of the present stock is sold until the additional furniture arrives, verify that the inventory on hand after the order is filled is described by the matrix $2M$. If the inventory at the end of June is described by the matrix

$$A = \begin{pmatrix} 5 & 3 & 1 & 2 \\ 6 & 2 & 1 & 5 \\ 1 & 0 & 3 & 3 \end{pmatrix},$$

interpret $2M - A$. How many suites were sold during the June sale? (10%)

3. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 1 & 2 & 0 \end{bmatrix}.$$

Let C be the unit square with vertices $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Find the area of the image of C under the transformation $x \rightarrow Ax$. Verify that the area equals vol(A). (10%)

4. For $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \in M_{2\times 2}(R)$, find an expression for A^n, where n is an arbitrary positive integer. (10%)

5. Suppose that $T: R^2 \rightarrow R^2$ is linear, $T(1,0) = (1,4)$, and $T(1,1) = (2,5)$. What is $T(2,3)$? Is T one-to-one? (10%)

6. Let the reduced row echelon form of A be

$$\begin{bmatrix} 1 & 0 & 2 & 0 & -2 \\ 0 & 1 & -5 & 0 & -3 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix}.$$

Determine A if the first, second, and fourth columns of A are $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, respectively. (14%)

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7. Let V be a vector space over F, where $F = R$ or $F = C$, and let W be an inner product space over F with inner product $\langle \cdot, \cdot \rangle$. If T: $V \rightarrow W$ is linear, prove that $(x, y)' = (T(x), T(y))$ defines an inner product on V if and only if T is one-to-one. (20%)

8. For each linear operator T on an inner product space V, determine whether T is normal, self-adjoint, or neither. If possible, produce an orthonormal basis of eigenvectors of T for V and list the corresponding eigenvalues.
(a) $V = R^2$ and T is defined by $T(a, b) = (2a - 2b, -2a + 5b)$. (8%)
(b) $V = R^3$ and T is defined by $T(a. b, c) = (-a + b, 5b, 4a - 2b + 5c)$. (8%)

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