1. **(30 points)** Consider the following primal linear programming problem (P).

$$
\text{(P)} \quad \text{Minimize } z = c_1x_1 + x_2
$$
subject to
$$
\begin{aligned}
-2x_1 - x_2 + x_3 \quad &= -6 \\
x_1 + 2x_2 \quad + x_4 &= \ \ 6 \\
x_1, x_2, x_3, x_4 &\geq 0
\end{aligned}
$$

(a) (5 points) If $c_1 = 1$, find an intial feasible solution of (x_1, x_2, x_3, x_4) by using Phase-I of the two-phase method.

(b) (5 points) If $c_1 = 1$, find an optimal solution of (x_1, x_2, x_3, x_4) by using the **simplex** method.

(c) (5 points) Write the dual problem.

(d) (5 points) If $c_1 = 1$, what is the optimal solution to the dual problem and its corresponding objective value of the dual problem?

(e) (5 points) If $c_1 = 1$, find an optimal solution of (x_1, x_2, x_3, x_4) to the promal problem (P) starting from a trivial **in**feasible solution, $(x_1, x_2, x_3, x_4) = (0, 0, -6, 6)$ by using the **dual simplex** method.

(f) (5 points) Construct the allowable range of c_1 that the optimal solution obtained in (b) stays optimal.

2. **(20 points)** Suppose you will choose 10 dishes from four meal types (pork, beef, vegitable, and grain). At least one dish from each meal type must be chosen. The 10 dishes are allocated to the four meal types in a manner that maximizes "nutrition." You measures nutrition on a 100-point scale as shown in the following table.

	Number of dishes chosen						
Meal types	1	2	3	4	5	6	≥7
Pork	25	50	60	80	100	100	100
Beef	20	70	90	100	100	100	100
Vegitable	40	60	80	100	100	100	100
Grain	10	20	30	40	50	60	70

How should you choose the dishes? Please answer the following questions.

(a) (5 points) What are the stages and states for the dynamic programming formulation of this problem?

(b) (15 points) Use dynamic programming to solve this problem.

3. **(30 points)** Consider the following optimization problem

$$
\min_{(x,y)\in R^2} f(x,y) = x^2 + 2xy + 2y^2 + 4x + 6y + 5
$$

$$
\text{subject to} \quad g_1(x,y) = 3 - x - y \leq 0, \ \ g_2(x,y) = -y \leq 0 .
$$

Answer the following questions.

(a) (6 points) Show that $f(x,y)$ is convex.

(b) (6 points) Write down the Karush-Kuhn-Tucker (KKT) conditions necessary for solving this optimization problem.

(c) (6 points) Continuing with (b), can you find a feasible solution when both constraints g_1 and g_2 are "inactive"? Show the details.

(d) (6 points) Continuing with (b), identify the feasible solution(s) (x^*, y^*) and check whether or not the Linear Independence Constraint Qualification (LICQ) holds at (x^*, y^*).

(e) (6 points) Find the global minimizer and the optimal value of $f(x,y)$.

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4. **(20 points)** There is a small YouBike station on the NTU campus, which has a maximum capacity of 2 bike docks. Suppose time is divided into discrete slots 1, 2, ..., where at any point in time, the following 2 random events may occur:

A bike is returned with probability $p = 0.4$,

A bike is rented with probability $q = 0.6$,

Define the state of the system as the number of bikes on the docks, then it can be modeled as a discrete-time Markov chain. Answer the following questions.

(a) (5 points) Identify the state space and the transition probability matrix of this Markov chain.

(b) (5 points) When a student arrives at the Youbike station to rent a bike, what is the probability that both docks are empty? (**Hint**: You can solve the steady-state probabilities from the flow balance equations.)

(c) (5 points) If a student arrives at the Youbike station, how many bikes does he expect to see on the docks?

(d) (5 points) A manager of YouBike company comes to observe the operation of this particular station. When he arrives at the station, both docks are empty. What is the expected number of time units until 2 bikes will be on the docks?

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