

Problem (1) (25%)

This is a 2-dimensional problem. As shown in Fig. 1a, water flows from a big reservoir through a gate and into a stream. Both far upstream (with flow depth H) and far downstream (with flow depth h), the flow condition is steady uniform.

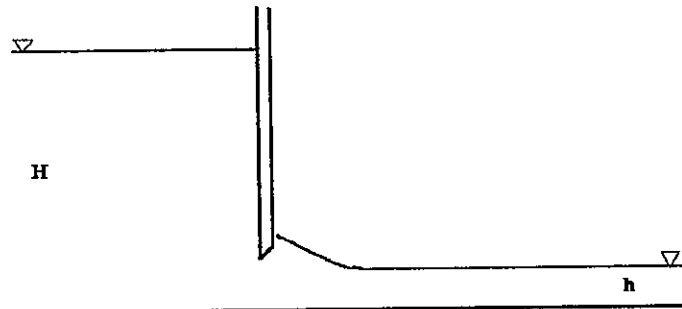


Fig. 1a

- (a) (10%) If the system is adiabatic, has no shaft energy input and no friction loss but do suffer energy loss when going through the gate with the energy head loss expressed as

$$h_L = 5 \frac{v^2}{2g}, \text{ with } v \text{ the downstream velocity, what are the relation between } H, \text{ and } h.$$

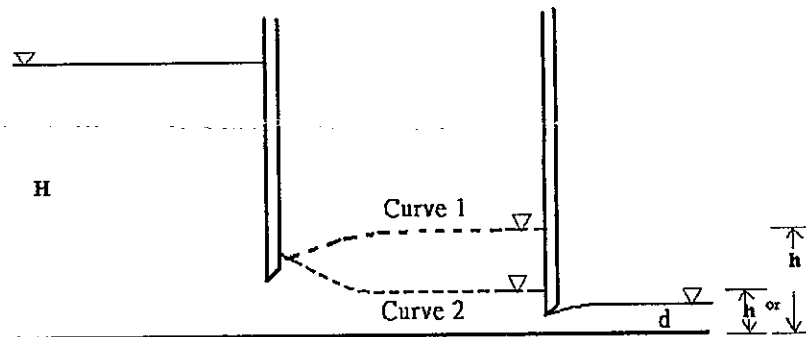


Fig. 1b

- (b) (5%) Now, as depicted in Fig.1b, there is another gate far downstream. If we lower the gate so that the gap is less than the downstream steady flow depth, what will happen to the flow? Describe the changes and then explain which of the two curves (curve 1 and curve 2) in Fig. 1b maybe the better possible final profile when the steady state reached again.
- (c) (10%) If h is the depth just behind the second gate, using the same assumption as in (a), calculate the relation between H , h and d .

Problem (2) (31%) In plain Cartesian Coordinate (x,y) , a Stream function $\phi = x^2 y$ is given

- (1) (5%) Is this an irrotational flow field? Calculate vorticity and then prove your answer.
- (2) (5%) Prove that x and y axis are both streamlines
- (3) (6%) Draw at least 3 stream lines in (x,y) plane. Casual draft will not be counted.
- (4) (5%) Calculate the acceleration field for this flow field
- (5) (5%) If y axis is a wall and pressure is P at the origin, derive the pressure distribution along y axis.
- (6) (5%) Streamlines cannot cross. However, there are streamlines meet at the origin. Does this imply that origin is a singular point? If your answer is YES, then explain why there is such a singular point. If your answer is NO, explain why streamlines intersect.

Problem (3) (26%) A rectangular tube connected to a quarter circular tube as shown in Fig. 3.

- (a) (10%) Water is stationary (no hole) in the tube. Calculate the total force (the magnitude and direction of the total force).on
- (1) Surface ABCD
 - (2) Surface EFCD
- (b) (16%) If there is a small hole at the midpoint of CD. The round hole has diameter 1cm.
- (1) Calculate the velocity and flow rate of water through the hole
 - (2) How long does it take for free surface to lower to height of AB?

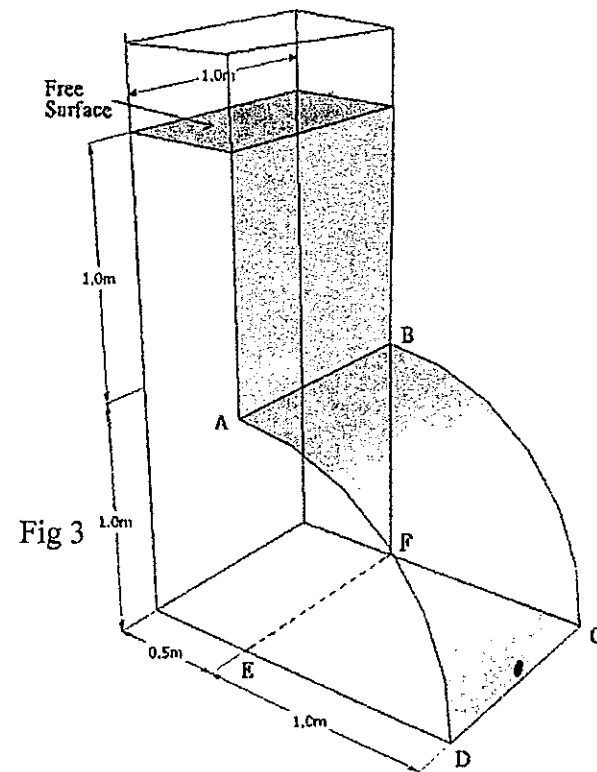


Fig 3

Problem 4. (18%)

A bridge is to be built on a wide river. The diameter of the cylindrical piers is 1m. The distance between the centers of adjacent piers is 10m. The river width is 500m and flow depth is 10m. The flow velocity is 1.0m/s. Now we use a length ratio 100 to build a model in laboratory. In the laboratory experiment, we consider only one pier; the setup is depicted in Fig. 2. The fluid used is mercury ($\rho_{\text{mercury}}/\rho_{\text{water}}=13.6$, and $v_{\text{mercury}}/v_{\text{water}}=2$). The measured impact force on the pier in lab is 5NT for the whole model pier.

- (a) (3%) List all the variables involved in this physical phenomenon.
- (b) (5%) Find all the dimensionless parameters (List the process)
- (c) (5%) What is the flow velocity in the experiment?
- (d) (5%) What is the impact force for the real pier?