Instructions:

- This exam has two parts.
  Part I consists of fill-in-the-blank problems. Only labeled final answers will be graded.
  Part II consists of partial credit problems. Any answer without explanation will not receive credit.
- No electronic devices or computer algebra systems allowed for this exam.
- Usage of any theorem/formula must be clearly stated.

Part I: 5 points for each blank.

(1) Evaluate $\lim_{x\to 0}\dfrac{\sqrt{9+x\sin x}-\sqrt{9+x^3}}{1-\cos(2x)}=$ __(1)__ .

(2) Let $y=f(x)$ be a function defined implicitly by the equation $\sqrt{2x}+\sqrt{3y}=xy-1$ near $x=2, y=3$. Evaluate $f'(2)=$ __(2)__ .

(3) Let $\mathcal{R}$ be the region described by $\{(x,y)\mid y^2\le x\le e^{y-1},\ 0\le y\le 1\}$. The volume obtained by rotating $\mathcal{R}$ about the x-axis is __(3)__ .

(4) The arc length of the curve $y^2=x^3$ from $(0,0)$ to $(4,8)$ is __(4)__ .

(5) Solve the initial value problem.

$$f'(x)-(\tan x)f(x)=\sin^2 x\ ,\quad f(0)=2$$

Then evaluate $f(\pi/4)=$ __(5)__ .

(6) The third-degree Taylor polynomial $T_3(x)$ for the function $f(x)=\tan^{-1}(x)$ centered at $a=1$ is __(6)__ .

(7) The maximum value of $f(x,y,z)=x^3yz$ subject to the constraint $x^2+y^2+z^2\le 4$ is __(7)__ .

(8) Evaluate $\int_0^1\int_x^{\sqrt{2-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} z^2\,dz\,dy\,dx=$ __(8)__ .

Part II: 20 points for each problem.

(9) Sketch the curve $y=\dfrac{x^2-2}{e^x}$. Label ALL asymptotes, local maxima/minima, inflection points on your sketch. You must justify each and every one of your answers.

(10) Evaluate the line integral

$$\oint_C (y+e^x)\,dx+(2x+\cos y)\,dy\ ,\quad C \text{ is the boundary of the region enclosed by the parabolas } y=x^2\ ,\ x=y^2.$$

by TWO methods: (a) Directly with parametrization of the curves, and (b) Using Green's Theorem, you should write down the statement of Green's Theorem.

(11) Let R be the region in the first quadrant bounded by the curves: $y=x$, $y=3x$, $xy=1$, $xy=3$.

(a) Find the Jacobian of the transformation: $x=u/v$, $y=uv$.

(b) Use the transformation in (a) to evaluate $\iint_R xy^3\,dA$.

(c) Evaluate the integral with a different transformation given by $U=xy$, $V=y/x$.

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