1. Find the dimension and the bases of the following homogeneous system. (10%)

$$x+2y+3z+t=0$$
$$2x+4y+7z+4t=0$$
$$3x+6y+10z+5t=0$$

2. Find a basis for the subspace W of R^4 that is orthogonal to u=(1,−2,3,4) and v=(3,−5,7,8). (10%)

3. Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis (u_1, u_2, u_3) into an orthonormal basis, where $u_1=(1,1,1)$, $u_2=(-1,1,0)$, $u_3=(1,2,1)$. (10%)

4. Let $T: R^2 \to R^3$ be the linear transformation defined by $T(a_1,a_2)=(a_1+3a_2, 0, 2a_1-4a_2)$. Let β and γ be the standard ordered bases for R^2 and R^3, respectively. Now $\beta=\{(1,0),(0,1)\}$ and $\gamma=\{(e_1,e_2,e_3)\}$. Find the matrix representation of T in the ordered bases β and γ $[T]_\beta^\gamma$. (10%)

5. Let $T: R^2 \to R^3$ be the linear transformation defined by $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)=\begin{bmatrix}x_2\\-5x_1+13x_2\\-7x_1+16x_2\end{bmatrix}$. Find the matrix for the transformation T with respect to the bases $B=(u_1,u_2)$ for R^2 and $B'=(v_1,v_2,v_3)$ for R^3, where $u_1=\begin{bmatrix}3\\1\end{bmatrix}$, $u_2=\begin{bmatrix}5\\2\end{bmatrix}$, $v_1=\begin{bmatrix}1\\0\\-1\end{bmatrix}$, $v_2=\begin{bmatrix}-1\\2\\2\end{bmatrix}$, $v_3=\begin{bmatrix}0\\1\\2\end{bmatrix}$. (15%)

6. Let $T: P_2 \to P_2$ be defined by $T(a_0+a_1x+a_2x^2)=(5a_0+6a_1+2a_2)-(a_1+8a_2)x+(a_0-2a_2)x^2$.
   (a) Find the matrix T with respect to the standard basis for P_2. (5%)
   (b) Find the eigenvalues of T. (5%)
   (c) Find bases for the eigenspaces of T. (5%)

7. Define $T: R^2 \to R^2$ $T(w,z)=(z,w)$. Find all the eigenvalues and eigenvectors of T. (15%)

8. In R^4, let $U=span((1,1,0,0),(1,1,1,2))$. Find $u \in U$ such that $\|u-(1,2,3,4)\|$ is as small as possible. (15%)

試題隨卷繳回