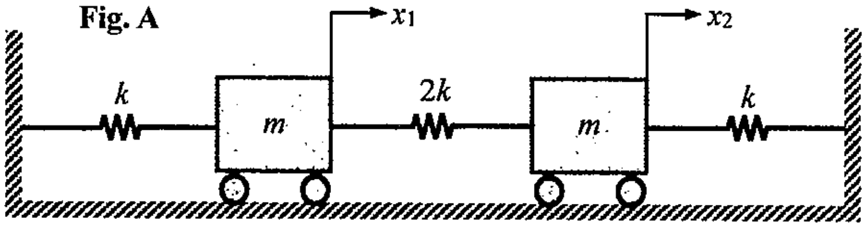
請於答案卷上作答，於試題卷上作答者，不予計分。

1. Consider the two-degrees-of-freedom mechanical system shown in **Fig. A**. The displacements x_1 and x_2 are measured from the respective equilibrium positions. Assume that two masses (m) move without friction. Determine the natural frequencies and modes of vibrations. To find the natural frequencies of the free vibration, assume that the motion is harmonic, i.e., assume that $x_1 = A\sin\omega t$ and $x_2 = B\sin\omega t$, where A and B are vibration amplitudes.【計分：20 分】

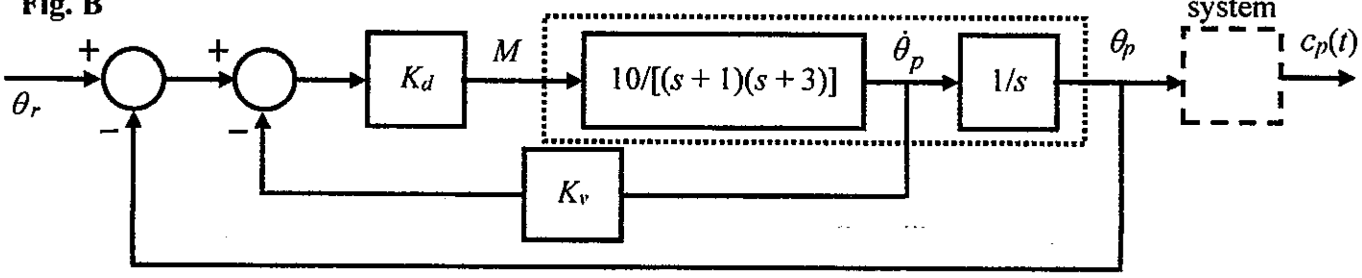
Fig. A



2. Consider the system defined by $\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2u_s(t) \\ 5u_s(t) \end{bmatrix}$ and $\mathbf{x}(0) = \mathbf{0}$, where $u_s(t)$ is the unit-step function. Obtain the response $\mathbf{x}(t)$ analytically.【計分：20 分】

3. The block diagram of the servo-control system for one of the axes of a digital plotter is shown in **Fig. B**. The input θ_r is the output of a digital computer, and the output θ_p is the position of the servomotor shaft. It is assumed that the pen-positioning system connected to the motor shaft is rigid (no dynamics) within the system bandwidth. (1) If $K_d = 1$, find the range of K_v for which the system is stable. (2) If $K_v = 0.3$, find the range of K_d for which the system is stable. (3) Consider that K_d is plotted along the x-axis (abscissa) and K_v along the y-axis (ordinate) of a plane (called the parameter plane). Show the regions of this plane, in which the system is stable. (4) Let $K_d = 1$. Find both the value of K_v that makes the system marginally stable and the period of the resulting oscillation.【計分：20 分，每一小題 5 分】

Fig. B



4. The block diagram of an electric train control is shown in **Fig. C**. The system parameters and variables are: $e_r(t)$ = voltage representing the desired train speed; $v(t)$ = speed of train, ft/sec; M =Mass of train = 30,000 lb/sec; K = amplifier gain; K_t = gain of speed indicator = 0.15 V/ft/sec. To determine the transfer function of the controller, we apply a step function of 1 V to the input of the controller, that is, $e_c(t) = u_s(t)$. The output of the controller is measured and described by the following equation:

$$f(t) = 100\left(1 - 0.3e^{-6t} - 0.7e^{-10t}\right)u_s(t)$$

(a) Find the transfer function $G_c(s)$ of the controller. (b) Derive the forward-path transfer function $V(s)/E(s)$ of the system. The feedback path is opened in this case. (c)Derive the closed-loop transfer function $V(s)/E_r(s)$ of the system. (d) Assuming that K is set at a value so that the train will not run away (unstable), find the steady-state speed of the train in feet per second when the input is $e_r(t) = u_s(t)$ V.【計分：20 分，每一小題5分】

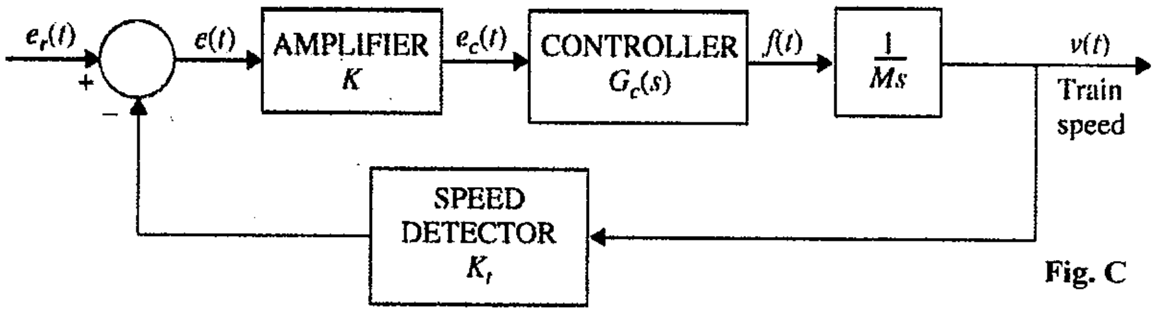


Fig. C

5. Given a dynamic system: $\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}u$, $y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\mathbf{x}$, where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is the state vector, u is the input, and y is the output of the system. Find the unit-step response of the dynamic system assuming zero initial condition.【計分：20分】

試題隨卷繳回