Problem 1. (20%)

Write down the equations of the following dimensionless groups or physical laws for transport phenomena.

(a) Reynolds number (for a circular tube with a diameter D). (4%)

(b) Newton's law of viscosity. (4%)

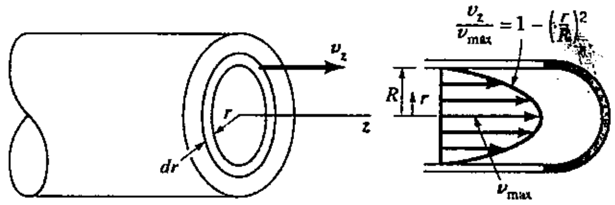
(c) Fourier's law of heat conduction. (4%)

(d) Newton's law of cooling. (4%)

(e) Fick's first law. (4%)

Problem 2. (20%)

Consider an incompressible Newtonian fluid flowing through a horizontal circular tube as illustrated below.



(a) The velocity profile $v_z(r)$ can be determined by solving the following partial differential equation (PDE).

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

Describe the name of this PDE and its physical meaning. (5%)

(b) Show that the velocity profile of v_z can be expressed as the following equation. (5%)

$$v_z = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)(r^2 - R^2)$$

(c) The volume rate of flow Q can be predicted by the Poiseuille law as written below. n = ? m = ? (4%)

$$Q = \frac{\pi R^n \Delta p^m}{8\mu L}$$

(d) Describe the criterion of the Reynolds number range for that Poiseuille's law is valid. (3%)

(e) Comment why Poiseuille's law is very helpful for cardiovascular disease prevention. (3%)

Problem 3. (10%)

(a) Draw a typical C_D (drag coefficient) *vs.* log Re (Reynolds number) plot for flow past an immersed spherical object for $10^{-3} < Re < 10^6$, and briefly justify your plot. (5%)

(b) Below is the Ergun equation. Briefly explain what Φ_s, D_p, and ε are, respectively. (5%)

$$\frac{\Delta p}{L} = \frac{150\bar{V}_0\mu}{\Phi_s^2 D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75\rho\bar{V}_0^{\,2}}{\Phi_s D_p}\frac{1-\varepsilon}{\varepsilon^3}$$

Problem 4. (30%)

The heat flux at the surface ($q_s = q_{x=0}$) for a semi-infinite unsteady-state 1D heat conduction in a solid can be described by the following empirical equation,

$$q_{s(x=0)} = A\frac{k}{\sqrt{\alpha t}}(T_\infty - T_s)$$

where T_∞ and T_s are the bulk and surface temperatures, respectively; A is an experimental constant.

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(a) Write down the SI units of k and α. (4%)

(b) Apply the dimensional analysis and the Fourier law of heat conduction to show the equation. (6%)

(c) Assume that q_s is resulted from a fluid convection process (*e.g.*, apply a hot air stream for solid heating). Comment under what kind of a Biot (Bi) number (high or low), the above equation is important and significant and explain why. (6%)

(d) Use the combined variable method to rigorously solve the temperature profile, as described by the following equation, for the heating a semi-infinite slab problem. (10%)

$$\frac{T - T_0}{T_1 - T_0} = 1 - erf\frac{x}{\sqrt{4\alpha t}}$$

(e) Propose a bioprocess scenario that can be described by the above heat-transfer model. (4%)

Problem 5. (20%)

(a) Consider a reduction reaction of O + ne$^-$ → R occuring at a planar platinum electrode. The current response under diffusion control (by applying a suffiently negative potential) can be expressed by the Cottrell equation as follows (assume there is no R initally).

$$i_d(t) = \frac{nFAD_O^{1/2}C_O^*}{\pi^{1/2}t^{1/2}}$$

where n, F, A, D_O, and C_O^* are the electron-transfer number, Faraday's constant, electrode area, diffusivity of O, and bulk molar concentration of O (*i.e.*, $C_O(x) = C_O^*$ = constant for the x position in solution far from the electrode surface), respectively. It is known that i_d(t) can be correlated to the molar flux of O at the electrode/solution interface ($J_O(x)$ at x = 0) according to the Faraday's law of electrolysis. Try your best to build an unsteady-state diffusion model with a governing PDE, initial condition, and boundary conditions for derivation of the Cottrell equation, and briefly describe the steps for determining i_d(t). (Note: assume one-dimensional diffusion, and you don't have to solve the PDE.) (7%)

(b) For l-D mass transfer of a charged species j, $J_j(x)$ (the molar flux of j) can be derived by the following equaiton.

$$J_j(x) = -\left(\frac{C_jD_j}{RT}\right)\left(\frac{\partial\bar{\mu}_j}{\partial x}\right) + C_jv(x)$$

where C_j, D_j, and v are molar concentration of j, diffusion coefficient of j, and flow velocity, respetively. And the electrochemical potential $\bar{\mu}_j$ for species j carrying a charge of z_j can be described below.

$$\bar{\mu}_j = \mu_j + z_jF\phi$$

in which μ_j is the chemical potential (partial molar Gibbs energy) of j, and ϕ is the electrical potential. Derive a generalized molar flux equation for $J_j(x)$ in terms of the graident of concentration of j, electric field, and flow velocity. (7%)

(c) According to (a), the Cottrell equation for a spherical electrode (with a radius r_0) is corrected as follows.

$$i_d(t) = nFAD_OC_O^*\left[\frac{1}{(\pi D_O t)^{1/2}} + \frac{1}{r_0}\right]$$

Explain why both (i) shriking the size of a working eletrode and (ii) introducing a capillary flow are effective methods to quickly attain a steady-state glucose biosensor current. Note that glucose is a neutral, non-charged molecule. (6%)

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