註：每題均需寫出詳細解題過程，僅寫出最終答案者不給分

1. (10 %) Suppose that y_1 is a solution of $y''+p(x)y'+q(x)y=0$. Find its second linearly independent solution y_2 in terms of y_1 and $p(x)$.

2. (15 %) Solve $(y')^2-xy'+y=0$.

3. (a) (5 %) Let $f(x,y,z)=x^2+y^2+2z^2$. Find the directional derivative of $f(x,y,z)$ at (1,1,1) in the direction of $\vec{c}=\vec{i}+\vec{j}+\vec{k}$.
   (b) (5 %) Find the unit inner normal of the surface $z^2=3(x^2+y^2)$ at (1,1,1).

4. (15 %) Find a matrix $\underline{X}$ which diagonalizes matrix $\underline{A}=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & -6 \end{bmatrix}$

5. (20%) Solve the following initial value problem using Laplace transform:

$$2\frac{d^2y}{dt^2}+t\frac{dy}{dt}-y=6 \qquad y(0)=y'(0)=0$$

6. (30%) Find the steady-state temperature u(r, z) in a cylinder defined by $0 \le r \le 1$ and $0 \le z \le 1$. The temperature is defined by the following equation:

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{du}{dr}+\frac{\partial^2 u}{\partial z^2}=0$$

The temperature u(r, z) is finite in the domain. The boundary conditions are given as follows:

$u(1,z)=kz,\quad 0<z<1$ (k is a constant)

$u_z(r,0)=0,\quad 0<r<1$

$u(r,1)=0,\quad 0<r<1$

試題隨卷繳回