※ 注意：請於試卷內之「非選擇題作答區」作答，並應註明作答之題號。

1. (8%) (7%) Let T be a failure time with support $\{t_1, t_2, \ldots, t_n, \ldots,\}$ and $\lambda_i \triangleq P(T = t_i | T \geq t_i)$, $i = 1, \ldots, n, \ldots$ Express $f(t_i) \triangleq P(T = t_i)$ and $S(t_i) \triangleq P(T > t_i)$ in terms of λ_j's, $i = 1, \ldots, n, \ldots$

2. (15%) Let X and Y be mutually independent and continuous random variables with the corresponding probability density functions $f_X(x)$ and $f_Y(y)$. Derive the probability density function of Y conditioning on $X - Y = 0$.

3. (7%) (8%) Let $X = R\cos\Theta$ and $Y = R\sin\Theta$, where R is a positive random variable on $(0, \infty)$, Θ is a uniform random variable on $(0, 2\pi)$, and R and Θ are mutually independent. Derive the corresponding distributions of X/Y and $2XY/\sqrt{X^2 + Y^2}$.

4. (15%) Let $U_1, \ldots, U_n, \ldots$ be independent random variables from Uniform$(0, 1)$ and $P(X = x) = (1/[(e-1)x!])I_{\{1,2,3,\ldots\}}(x)$ be the probability density function of X. Find the probability density function of $Z = \min\{U_1, \ldots, U_X\}$.

5. (7%) (8%) Let $X_1, \ldots, X_n$ be a random sample from Uniform$(\theta, \theta + 1)$. Find a minimal sufficient statistic of θ and derive its distribution.

6. (15%) Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$, where μ and σ^2 are unknown parameters. Derive the power function of the size α likelihood ratio test for the hypotheses $H_0 : \mu \leq \mu_0$ versus $H_A : \mu > \mu_0$.

7. (10%) Let $X_1, \ldots, X_n$ be a random sample from a geometric distribution $P(X = x) = p(1-p)^{x-1}I_{\{1,2,\ldots\}}(x)$ and p have a uniform prior distribution on $(0, 1)$. Find the Bayes estimator of p based on the loss function $L(p, \delta(X_1, \ldots, X_n)) = (\delta(X_1, \ldots, X_n) - p)^2$

試題隨卷繳回