Consider a 1-dimensional simple harmonic oscillator composed of a spring of spring constant k and a point mass m. Answer the following questions.　※ 注意：請於試卷內之「選擇題作答區」依序作答。 每題 5 分

1. What is its natural frequency? (A) $f = \frac{1}{2\pi}\sqrt{m/k}$ (B) $f = 2\pi\sqrt{m/k}$ (C) $f = \frac{1}{2\pi}\sqrt{k/m}$ (D) $f = 2\pi\sqrt{k/m}$ (E) $f = \sqrt{m/k}$

2. Following the previous question, what is the Lagrangian of the simple harmonic oscillator? (A) $L = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ (B) $L = \frac{1}{2}mv^2 - \frac{1}{2}kx^2$ (C) $L = \frac{1}{2}mv^2 + \frac{1}{2}kx$ (D) $L = \frac{1}{2}mv^2 - \frac{1}{2}kx$ (E) $L = mv^2 + kx$

3. Which of the following is correct? (A) Given physical quantities A and B of the same dimension (e.g. they are both the lengths of something), and quantities C and D of the same dimension, if D is uniquely determined by A, B, and C, there must be some numerical constants k and n (of no dimension) such that $D = kC(A/B)^n$. (B) The moon is orbiting around the earth with roughly the same period as that for its spinning around its own axis. (C) The gravitational potential energy for N point masses M_i is given by $U = -\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{GM_iM_j}{R_{ij}}$. where R_{ij} is the distance between particle i and particle j, and G is the Newton constant. (D) As the kinetic energy is given by a different formula, the total energy of an isolated system is not conserved in special relativity. (E) The friction is a conservative force.

A uniform ruler of length L and mass M is placed on a frictionless surface, initially at rest along the y-axis. A small piece of clay of mass $M/2$ moving at the speed V in the x-direction hits the top of the ruler and sticks to it. (See Fig.A below.) Consider the ruler and the clay as a single system, and answer the following questions.

4. What is the center-of-mass velocity of the system? (A) $V_{CM} = V/2$ (B) $V_{CM} = V/3$ (C) $V_{CM} = 3V/2$ (D) $V_{CM} = 2V/3$ (E) $V_{CM} = V/6$

5. What is the moment of inertia I of the system around its center-of-mass? (A) $I = \frac{3}{2}ML^2$ (B) $I = \frac{2}{3}ML^2$ (C) $I = \frac{1}{6}ML^2$ (D) $I = \frac{7}{15}ML^2$ (E) $I = \frac{11}{18}ML^2$

6. What is the magnitude of the total angular momentum of the system around its center of mass? (A) $\frac{1}{3}MVL$ (B) $\frac{2}{3}MVL$ (C) $\frac{1}{6}MVL$ (D) $\frac{2}{9}MVL$ (E) $\frac{7}{18}MVL$

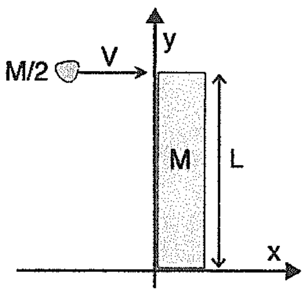

Fig. A

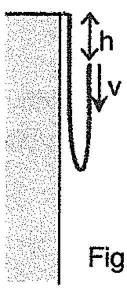

Fig. B

7. Which of the following is wrong? (A) Divide a uniform spring of spring constant k into two identical halves. The spring constant of each half is 2k. (B) Fix one endpoint of a rope on the edge of a cliff, and release the other endpoint from equal height. At the moment when the free endpoint falls a distance h below the other endpoint, its velocity v is smaller than $\sqrt{2gh}$ for any h > 0. (See Fig. B above.) (C) Newton's 3rd law is necessary for the momentum conservation of

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physical systems. (D) Given arbitrary 3-dimensional vectors A, B, and C, the following identity holds: A×(B×C) = B(A•C) – C(A•B) (E) The angular momentum of a point mass in linear motion at constant velocity can be non-zero.

8. The change in entropy for an ideal gas over a reversible isothermal process at the temperature *T* from the initial pressure *P* and volume *V* to the final pressure *P/2* and volume *2V* is (A) *PV log(1/2)* (B) *PV log(2)* (C) *PV log(1/2)/T* (D) *PV log(2)/T* (E) *2PV log(2)/T*

9. Which of the following statement is correct? (A) The 0th law of thermodynamics states that two systems in thermal contact always reach thermal equilibrium in the end. (B) The entropy of any macroscopic system never decreases over time. (C) The entropy of an ideal gas is doubled if its volume is doubled at the same temperature. (D) Any irreversible engine always has a smaller efficiency than any reversible engine. (E) By choosing a suitable definition of Kelvin, the Boltzmann constant has an exact value that does not have to be determined by experiments.

10. For a string of length *L* with one fixed end and one open end, the oscillation wave on the string travels at the speed *v*. What are the standing wave frequencies? (A) f = v/L, v/2L, v/3L, … (B) f = v/L, 2v/L, 3v/L, … (C) f = 2v/L, v/L. 3v/2L, … (D) f = v/2L, 3v/2L, 5v/2L, … (E) f = v/4L, 3v/4L, 5v/4L, …

11. Which of the following describes "no existence of magnetic monopole."?

(A) $\nabla \times \vec{B} = 0$

(B) $\nabla \times \vec{B} = \mu_0 \left(\vec{J}_{in} + \frac{d\vec{E}}{dt}\right)$

(C) $\nabla \cdot \vec{B} = 0$

(D) $\nabla \cdot \vec{B} = \mu_0 \left(\vec{J}_{in} + \frac{d\vec{E}}{dt}\right)$

(E) none of above

A charge Q is distributed over a metallic sphere of radius "a". Please calculate the following:

12. Capacitance (C): (A) $2\pi\varepsilon_0 a$ (B) $\pi\varepsilon_0 a$ (C) $4\pi\varepsilon_0 a$ (D) $8\pi\varepsilon_0 a$ (E) $\pi\varepsilon_0 a/2$

13. Potential energy saved in this system (U_E): (A) $\frac{Q^2}{4\pi\varepsilon_0 a}$ (B) $\frac{Q^2}{8\pi\varepsilon_0 a}$ (C) $\frac{Q^2}{2\pi\varepsilon_0 a}$ (D) $\frac{Q^2}{16\pi\varepsilon_0 a}$ (E) none of above.

14. What is the electric field at a distance r from an infinitely long string uniformly charged with linear charge density λ ?

(A) $\frac{\lambda}{2\pi\varepsilon_0 r}$ (B) $\frac{\lambda}{\pi\varepsilon_0 r}$ (C) $\frac{\lambda}{4\pi\varepsilon_0 r}$ (D) $\frac{2\lambda}{\pi\varepsilon_0 r}$ (E) none of above.

15. The uniform surface charge density on an infinitely long strip of finite width *2a* is σ. Calculate the electric field at a distance b from the center of the metallic plane, as the point P in Fig. 1.

(A) $\frac{\sigma}{\pi\varepsilon_0}tan^{-1}\left(\frac{a}{b}\right)$ (B) $\frac{\sigma}{\pi\varepsilon_0}cot^{-1}\left(\frac{a}{b}\right)$ (C) $\frac{\sigma}{\pi\varepsilon_0}tan^{-1}\left(\frac{b}{a}\right)$ (D) $\frac{\sigma}{\pi\varepsilon_0}cot^{-1}\left(\frac{b}{a}\right)$ (E) none of the above.

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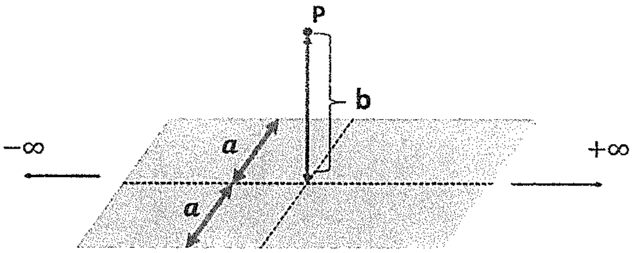

Fig. 1

A disk of radius R has a total charge q uniformly distributed on it. The surface charge density of the disk is σ. The disk is rotating around a vertical axis through its center with angular frequency w. Please calculate the following:

16. The magnetic field B at the center of the rotating disk (point P in Fig. 2)

(A) $\frac{\mu_0 wq}{4\pi R}$ (B) $\frac{2\mu_0 wq}{3\pi R}$ (C) $\frac{\mu_0 wq}{2\pi R}$ (D) $\frac{\mu_0 wq}{\pi R}$ (E) none of above

17. The magnetic field B at distance x from the center of the disk (point Q in Fig. 3)

(A) $\frac{\mu_0 \sigma w}{2}[\sqrt{R^2+x^2}+\frac{x^2}{\sqrt{R^2+x^2}}-3x]$ (B) $\frac{\mu_0 \sigma w}{2}[\sqrt{R^2+x^2}+\frac{x^2}{\sqrt{R^2+x^2}}-2x]$

(C) $\frac{\mu_0 \sigma w}{2}[\sqrt{R^2+x^2}+\frac{x^2}{\sqrt{R^2+x^2}}-x]$ (D) $\frac{\mu_0 \sigma w}{2}[\sqrt{R^2+x^2}+\frac{x^2}{2\sqrt{R^2+x^2}}-x]$

(E) none of above

18. Effective magnetic moment of this rotating disk

(A) $\frac{3\sigma w\pi R^4}{4}$ (B) $\frac{\sigma w\pi R^4}{4}$ (C) $\frac{2\sigma w\pi R^4}{3}$ (D) $\frac{\sigma w\pi R^4}{2}$ (E) none of above

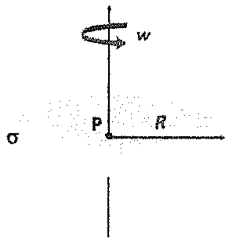

Fig. 2

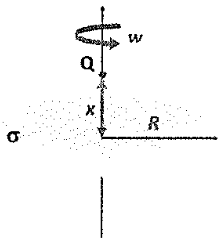

Fig. 3

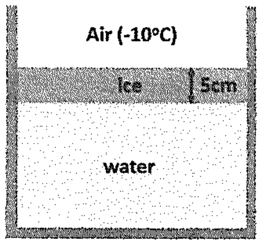

Fig. 4

A container of water has been outdoors in cold weather for a long time, and a slab of ice with a thickness of 5 cm has formed on its surface [See Fig. 4]. The air above the ice is at -10°C.

19. What is the water temperature? (A) 4 °C (B) -10 °C (C) 0 °C (D) -5 °C (E) unknown

20. Please calculate the rate of ice formation (in centimeters per hour) on the bottom of the ice slab. Take the thermal conductivity of ice to be 0.004 cal/s· cm · °C, the latent heat of fusion of water to be 80 cal/g, and density of ice to be 0.92 g/cm^3; here we assume no heat transfer through the tank walls or bottom. (A) 0.55 cm/hr (B) 0.39 cm/hr (C) 0.62 cm/hr (D) 0.24 cm/hr (E) 0.12 cm/hr

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