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國立臺灣大學 110 學年度碩士班招生考試試題

科目:近代物理學(A)

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※注意:

請於答案卷內之「選擇題作答區」作答,未正確填答於「選擇題作答區」不予計分。 選擇題為單選題,每題 5分,請依答案卷首頁所印題號序作答。計算過程可利用答案卷空白處書 寫,但不列入計分。

Some useful constants and conversion factors

Quantity	Symbol	Value
Speed of light	с	$3.00 \times 10^{8} \text{ m/s}$
Electron charge	e	$1.60 \times 10^{-19} \mathrm{C}$
Planck constant	h	$6.63 \times 10^{-34} \text{ J} \cdot \text{s}$
Reduced Planck constant	ħ	$1.05 \times 10^{-34} \text{ J} \cdot \text{s}$
Vacuum permittivity	ϵ_0	$8.85 \times 10^{-12} \text{ F/m}$
Mass of a proton	m_p	$1.67 \times 10^{-27} \text{ kg}$
Mass of an electron	m_e	$9.10 \times 10^{-31} \text{ kg}$

- 1. A train of proper length L_0 is moving at a velocity v=0.8c towards a tunnel of which the proper distance is also L_0 . At time t=0, the front end of the train meets the entrance of the tunnel viewed both from the ground and the train. Suppose the time when the rear end of the train meets the entrance of the tunnel viewed from the ground and the train is t_A and t'_A , respectively, and the time when the front end of the train meets the exit of the tunnel viewed from the ground and the train is t_B and t_B' . Which of the following is incorrect?
 - (A) In the frame of the ground, $t_A < t_B$.
 - (B) In the frame of the train, $t'_{B} < t'_{A}$.
 - (C) In the frame of the train, the duration from the train entering the tunnel and entirely get out of the tunnel is $\frac{8L_0}{5v}$.

(D)
$$t_{\rm B} = \frac{L_0}{v}$$
 and $t_{\rm B}' = \frac{4L_0}{5v}$.
(E) $t_{\rm A}' = t_{\rm B}$.

- 2. A Rydberg atom is one whose excited state is of very large principal number n. When n is large, the energy level spacing is proportional to
 - (A) n (B) $\log n$ (C) 1/n (D) $1/n^2$ (E) $1/n^3$.
- 3. A positronium is an exotic atom consisting of an electron and its anti-particle, a positron, bound together. Its ground state energy is

(A)
$$-3.2 \text{ eV}$$
 (B) -6.8 eV (C) -9.4 eV (D) -11.4 eV (E) -13.6 eV .

- 4. A particle of rest mass 1 MeV has a momentum 7.1×10^{-22} kg·m/s. The kinetic energy is (A) 0.25 MeV (B) 0.33 MeV (C) 0.50 MeV (D) 0.67 MeV (E) 1.2 MeV.
- 5. The ground state of a harmonic oscillator is $\psi(x) = \frac{1}{\pi^{2/4}\sqrt{x_0}}e^{-x^2/(2x_0^2)}$, where $x_0 = \sqrt{\frac{\hbar}{m\omega}}$. The corresponding uncertainty of the momentum observable $\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2}$ is $(A) \frac{\hbar}{\sqrt{2}x_0} (B) \frac{\hbar}{x_0} (C) \frac{\hbar}{2x_0} (D) \frac{2\hbar}{x_0} (E) \frac{\sqrt{2}\hbar}{x_0}.$
- 6. Following 5. Determine the wave function of the first excited state of a harmonic oscillator:

(A)
$$\frac{x}{\pi^{1/4}x_0^{3/2}}e^{-x^2/x_0^2}$$
 (B) $\frac{x}{\pi^{1/4}\sqrt{2}x_0^{3/2}}e^{-x^2/(2x_0^2)}$ (C) $\frac{\sqrt{2}x}{\pi^{1/4}x_0^{3/2}}e^{-x^2/(2x_0^2)}$ (D) $\frac{1}{\pi^{1/4}\sqrt{x_0}}\left(1 - \frac{2x}{x_0}\right)e^{-x^2/x_0^2}$ (E) $\frac{1}{\pi^{1/4}\sqrt{x_0}}\sin\left(\frac{x}{x_0}\right)e^{-x^2/(2x_0^2)}$.

7. A radioisotope X has a half life time of 10 years. A sample has an initial decay rate of 1.38×10^{13} decay/year. How many nuclei decay in 10 years? (ln 2 = 0.69) (A) 10^{11} (B) 10^{12} (C) 10^{13} (D) 10^{14} (E) 10^{15} .

見背面

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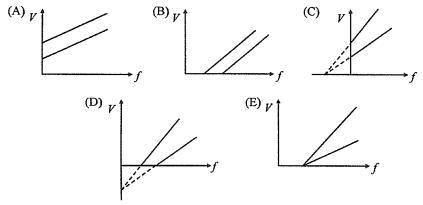
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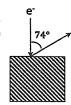
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8. Which of the following is incorrect:

- (A) The blackbody radiation displays a continuous spectrum.
- (B) The distribution of energy of the emitters obeys the equipartition of energy in each degree of freedom in blackbody radiation.
- (C) The radiated wavelength of maximal intensity is inversely proportional to the temperature.
- (D) The energy of the oscillator is quantized. It can have only certain discrete amounts of energy $E_n = nhf$, where n is a positive integer and f is the frequency of oscillation.
- (E) The correspondence principle states that quantum results must blend smoothly with classical results when the quantum number becomes large.
- 9. In photoelectric effect experiments, two different materials are illuminated by light of frequency f. The induced electrons passing a pair of parallel plates form a current that can be measured. The pair of plates are applied a voltage to decrease the current. The extinction voltage V is the minimal value of voltage such that the measured current becomes 0. Which of the following plots mostly represents the f-V relation?



- 10. In the Davisson-Germen experiment, a beam of 60 eV electrons is directed at a crystal, and diffracted electrons are found at an angle $\theta = 74^{\circ}$ relative to the original beam. What is the spacing of the atomic planes of the crystal?
 - (A) 0.1 nm (B) 0.5 nm (C) 1 nm (D) 2 nm (E) 5 nm.



- 11. How many wave functions are possible in the n = 5 shell? (A) 9 (B) 16 (C) 20 (D) 25 (E) 30.
- 12. A quantum system has a set of energy eigenstates $|E_i\rangle$. The system is in the state $|\psi\rangle = \frac{1}{\sqrt{30}}|E_1\rangle + \frac{2}{\sqrt{30}}|E_2\rangle + \frac{3}{\sqrt{30}}|E_3\rangle + \frac{4}{\sqrt{30}}|E_4\rangle$, where the energies are given by $E_n = nE_1$. What is the probability that a measurement of energy yields the value of $3E_1$?
 - (A) $3/\sqrt{30}$ (B) $9/\sqrt{30}$ (C) 1/10 (D) 3/10 (E) 9/10.
- 13. In an infinite square well, for which of the following states will the particle never be found in the center of the well?
 - (A) The ground state (B) The first excited state (C) the second excited state (D) Any of the above (E) None of the above.

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14. An electron is in an infinite square well of width a:

$$V(x) = \begin{cases} \infty, & x < 0 \text{ and } x > a \\ 0, & 0 < x < a \end{cases}$$

 $V(x) = \begin{cases} \infty, & x < 0 \text{ and } x > a \\ 0, & 0 < x < a \end{cases}$ If the electron is in the state $\psi(x) = A \sin \frac{2\pi x}{a}$, at what position is the probability function a maximum? (A) 0 (B) a/2 (C) a/3 (D) a/4 (E) a/5.

- 15. An electron is in an infinite square well of width a with the same potential form as in Problem 14, with a = 300 pm wide. What is the probability that one can detect the electron in the first excited state within an interval between x = 0.50a and x = 0.75a? (A) 0 (B) 1/2 (C) 1/3 (D) 1/4 (E) 1/6.
- 16. A system consists of 4 non-interacting electrons localized inside an infinite square well of width a. What is the total energy of this system at T = 0 K?

(A)
$$4\frac{\pi^2\hbar^2}{2ma^2}$$
 (B) $6\frac{\pi^2\hbar^2}{2ma^2}$ (C) $8\frac{\pi^2\hbar^2}{2ma^2}$ (D) $10\frac{\pi^2\hbar^2}{2ma^2}$ (E) $12\frac{\pi^2\hbar^2}{2ma^2}$

- 17. In the polar coordinate (ρ, θ, ϕ) , a particle is described by the wave function $\psi(\rho, \phi)$ $Ae^{-\rho^2/(2\Delta^2)}\cos^2\phi$. What is the probability that a measurement of L_z yields the value $2\hbar$ for this particle? (A)1/2 (B) 1/3 (C) 1/4 (D) 1/5 (E) 1/6.
- 18. A ruby laser consists mostly of alumina (Al₂O₃) and a small amount of chromium, which is responsible for the red color of the ruby. This laser emits a pulse of light ($\lambda = 685$ nm) with a power of 3.00 kW over a period of 10.0 ns. How many chromium atoms undergo stimulated emission to produce this pulse? (A) 1.03×10^{13} (B) 1.03×10^{14} (C) 1.03×10^{15} (D) 1.03×10^{16} (E) 1.03×10^{17} .
- 19. The rotational energy of the CO molecule is $E_{rot} = \frac{\hbar^2}{2J}J(J+1)$ with $J=0,1,2,\cdots$. The J=0 to J=1 transition occurs at a frequency of 1.18×10^{11} Hz. What is the moment of inertia of this molecule? (A) $1.46 \times 10^{-26} \text{ kg} \cdot \text{m}^2$ (B) $1.46 \times 10^{-36} \text{ kg} \cdot \text{m}^2$ (C) $1.46 \times 10^{-46} \text{ kg} \cdot \text{m}^2$ (D) $1.46 \times 10^{-56} \text{ kg} \cdot \text{m}^2$ (E) $1.46 \times 10^{-66} \text{ kg} \cdot \text{m}^2$.
- 20. The magnitude of the orbital angular momentum of an electron in a p state of hydrogen is about (A) 1.0×10^{-34} J·s (B) 1.5×10^{-34} J·s (C) 2.0×10^{-34} J·s (D) 2.5×10^{-34} J·s (E) 3.0×10^{-34} J·s.

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