

※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

1. (5%) Which are the following can be true for an ordinary differential equation?

- (A) One independent variable
- (B) More than one independent variable
- (C) One dependent variable
- (D) More than one dependent variable
- (E) None of the above

2. (5%) Which are the possible solutions of the differential equation $\frac{dy}{dx} = 3x^2 - 3$, where $y(0) = 0$?

- (A) $x = 1$
- (B) $x = -1$
- (C) $y = x^3 - 3x$
- (D) $y = \frac{1 - e^{6x}}{1 + e^{6x}}$
- (E) $y = \frac{1 + e^{6x}}{1 - e^{6x}}$

3. (5%) Which are the possible solutions of the differential equation $1 + e^{3x} \frac{dy}{dx} = 0$, where $y(0) = 0$.

- (A) $y = e^{3x} - 1$
- (B) $y = e^{-3x} - 1$
- (C) $y = \frac{1}{3}(e^{3x} - 1)$
- (D) $y = \frac{1}{3}(e^{-3x} - 1)$
- (E) $y = -\frac{1}{3}(e^{-3x} - 1)$

4. (5%) Which are the possible solutions of the differential equation $\frac{dy}{dx} + 2xy^2 = 0$, where $y(0) = 1$.

- (A) $y = \frac{1}{x^2 + 1}$
- (B) $y = \frac{1}{x^2 + x + 1}$
- (C) $y = \frac{1}{\sqrt{x^2 + 1}}$
- (D) $y = \left(\frac{1}{x^2 + 1}\right)^2$
- (E) $y = \left(\frac{x}{x^2 + 1}\right)^2$

5. (5%) Which is the general solution of the differential equation $\frac{dx}{dy} = e^{y+x}$? (assume C is a constant)

- (A) $y = e^y + e^x$
- (B) $e^y + e^x = C$
- (C) $e^{-y} + e^{-x} = C$
- (D) $e^{-y} + e^x = C$
- (E) $e^y + e^{-x} = C$

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6. (5%) Which are the possible solutions of the differential equation

$$\frac{d^2y}{dx^2} + y = 0, \text{ where } y(0) = 1 \text{ and } \frac{dy(0)}{dx} = 1?$$

- (A) $y = e^x$
 (B) $y = \sin(x) + \cos(x)$
 (C) $y = 2 - e^{-x}$
 (D) $y = x + 1$
 (E) $y = -x + 2e^x - 1$

7. (5%) Which is the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0? \text{ (assume } C_1 \text{ and } C_2 \text{ are constants)}$$

- (A) $y = C_1e^x$
 (B) $y = C_1e^x + C_2e^{-2x}$
 (C) $y = C_1e^{-3x} - C_2e^{2x}$
 (D) $y = C_1e^x + C_2e^{2x}$
 (E) $y = C_1e^{-3x} + C_2e^{2x}$

8. (5%) Which is the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 0? \text{ (assume } C_1 \text{ and } C_2 \text{ are constants)}$$

- (A) $y = C_1e^{5x}$
 (B) $y = C_1e^{-5x}$
 (C) $y = C_1e^{5x} + C_2e^{-5x}$
 (D) $y = C_1e^{5x} + C_2xe^{-5x}$
 (E) $y = C_1e^{5x} + C_2xe^{5x}$

9. (5%) Which are the possible solutions of function $f(t)$ for the Laplace Transform

$$F(s) = \mathcal{L}(f) = \frac{s-2}{s^2-4s+4+\pi^2}?$$

- (A) $f = \cosh(\pi t)$
 (B) $f = \sinh(\pi t)$
 (C) $f = e^{2t} \cos(\pi t)$
 (D) $f = e^{2t} \sin(\pi t)$
 (E) $f = e^{2t} - 4e^{\pi t}$

10. (5%) Which are the possible solutions of system of equations for $x(t)$ and $y(t)$, where $y(0) = 1$ and

$$\frac{dy(0)}{dx} = 1?$$

$$\begin{cases} \frac{d^2x}{dt^2} - \frac{dx}{dt} + \frac{d^2y}{dt^2} = -2 \sin(t) \\ \frac{d^2x}{dt^2} + \frac{dx}{dt} + \frac{d^2y}{dt^2} = 0 \end{cases}$$

- (A) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(t) \\ t + 1 \end{bmatrix}$

- (B) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \sin(t) \\ t + 1 \end{bmatrix}$
- (C) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$
- (D) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(t) \\ e^t \end{bmatrix}$
- (E) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \sin(t) \\ e^t \end{bmatrix}$

11. (10%) Let Q be an $n \times n$ matrix. Then which of the following set is not a subspace?

- (A) Col Q
- (B) Null Q
- (C) rank Q
- (D) Row Q
- (E) None of the above

12. (10%) Let M be a subset of R^3 containing two or more vectors. Then:

- (A) The span of any two vectors in M is a plane in R^3
- (B) Every vector in M is in the span of M
- (C) If M contains more than three vectors, then M is linearly independent
- (D) The span of any two nonzero vectors in M is a plane in R^3
- (E) None of the preceding statements is true

13. (10%) Which of the following statements is true for all $n \times n$ matrices Q ?

- (A) Q has n distinct eigenvalues
- (B) If one of the eigenvalues of Q has multiplicity greater than one, then Q has fewer than n eigenvalues
- (C) If Q has no eigenvalues, then the degree of its characteristic polynomial is zero
- (D) If one of the eigenvalues of Q has multiplicity greater than one, then Q has fewer than n eigenvectors
- (E) None of the preceding statements are true

14. (10%) Suppose that s , t , and u are vectors in G^n such that s is orthogonal to u and u is orthogonal to t . Then

- (A) For any orthogonal $n \times n$ matrix G , we have that Gs is orthogonal to u
- (B) For any orthogonal $n \times n$ matrix G , we have that Gt is orthogonal to both s and u
- (C) s is orthogonal to t
- (D) $s + t$ is orthogonal to u
- (E) None of the preceding statements are true

15. (10%) An $m \times n$ matrix X is invertible if

- (A) The columns of X span R_m
- (B) The reduced row echelon form of X is I_n
- (C) The rows of X are linearly independent
- (D) The columns of X are linearly independent
- (E) None of the preceding statements is true