Part I. Multiple-choice questions ※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

Note:

(1) Please choose only one of the answer choices (a)-(e).

(2) In the following questions, N(a, b) refers to a normal distribution with the mean equal to a and variance equal to b.

(3) <u>Write down your answers on scantron answer sheet.</u>

(4) Each question is worth 5 points.

(5) The table of standard normal cumulative probability is used for the questions of both Parts I and II.

1. Suppose X and Y have a bivariate normal distribution with covariance 1. X has a mean of 10 and standard deviation of 4. Y has a mean of 15 and standard deviation of 6. $Z = 2X + 3Y$. Which of the following statement is false?
   a. The mean of Z is 65.
   b. The variance of Z is 400.
   c. The correlation between X and Y is 1/24.
   d. Z follows a normal distribution.
   e. None of the above choices (a)-(d).

2. Y is distributed N(0,4), and X is distributed N(0,16). Please calculate $E(Y^2)$ and $E(X^3)$.
   a. $E(Y^2) = 4$ and $E(X^3) = 16$
   b. $E(Y^2) = 2$ and $E(X^3) = 64$
   c. $E(Y^2) = 4$ and $E(X^3) = 0$
   d. $E(Y^2) = 2$ and $E(X^3) = 8$
   e. None of the above choices (a)-(d).

3. A random draw (with replacement) of 1000 balls from a pool of several million balls is conducted. Each ball is either blue or red. Let p denote the fraction of blue balls in the population. Let $\hat{p}$ denote the fraction of blue balls in the sample. You want to test the competing hypotheses H_0: $p = 0.4$ vs. H_1: $p \neq 0.4$. Suppose that you decide to reject H_0 if $|\hat{p} - 0.4| > 0.02$. Note that the sample of 1000 balls is considered as a large sample. What is the type I error of this test?
   a. 0.197
   b. 0.261
   c. 0.325
   d. 0.413
   e. 0.522

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4. Suppose you estimate a sample OLS regression model using 20 observations (for $i = 1, 2, \dots, 20$). The resulted regression equation is: $Y_i = 2.11 + 3.58X_i + \varepsilon_i$. Y_i, X_i, and ε_i denote the dependent variable, independent variable, and the regression residual, respectively. If X_i is equal to 2 for $i = 1, 2, \dots, 4$; equal to 3 for $i = 5, 6, \dots, 10$; equal to 11 for $i = 11, 12$; equal to 14 for $i = 13, 14, \dots, 20$, what is the value of $\overline{Y}$?
   a. 30.75
   b. 28.96
   c. 26.12
   d. It cannot be determined.
   e. None of the above choices (a)-(d).

5. Suppose a population regression equation is: $Y_i = \beta_0 + \beta_1 X_i + u_i$. Y_i, X_i, and u_i denote the dependent variable, independent variable, and the error term, respectively. i indexes each individual observation. Which of the following statement is correct?
   a. If $E(u_i|X_i) \neq 0$ and the correlation between X_i and u_i is zero, the OLS estimator $\hat{\beta}_1$ is unbiased.
   b. If $E(u_i|X_i) \neq 0$ and the correlation between X_i and u_i is zero, the OLS estimator $\hat{\beta}_1$ is inconsistent.
   c. If $E(u_i|X_i) = 0$, the correlation between X_i and u_i may not be zero.
   d. The assumption that the correlation between X_i and u_i is zero cannot be tested empirically.
   e. None of the above choices (a)-(d).

Please answer questions 6 and 7 using the following information:

Equation 1: $\ln(\widehat{Earnings}) = 5.18 + 0.31 CollegeDegree.$

Equation 2: $\ln(\widehat{Earnings}) = 3.86 + 0.55 CollegeDegree + 0.37\, Age$
$n = 16{,}370\,,\ \bar{R}^2 = 0.442\,.$

$Earnings$ measures a worker's monthly earnings. $CollegeDegree$ is an indicator variable that is equal to 1 for workers who have a college degree, and 0 otherwise. Age measures a worker's age in years. We take a natural logarithm on $Earnings$ (measured in thousand dollars).

6. In Equation 2, holding all other factors constant, when Age increases by 1 year, what is the predicted change in Earnings?
   a. Increase by 370 dollars.
   b. Increase by 1448 dollars.
   c. Increase by 0.37%.
   d. Increase by 37%.
   e. None of the above choices (a)-(d).

7. Suppose Equation 2 is the correct model. That is, Age is a determinant of a person's earnings. Are older people more likely than younger people to have a college degree in this sample?
   a. Older people are more likely to have a college degree.
   b. Younger people are more likely to have a college degree.
   c. All people are equally likely to have a college degree.
   d. We don't have sufficient information to answer this question.
   e. None of the above choices (a)-(d).

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8. Which of the following is false?
   a. If all Gauss-Markov conditions are satisfied, the OLS estimators have the lowest conditional variance among all linear conditionally unbiased estimators.
   b. If not all Gauss-Markov conditions are satisfied, the OLS estimators are biased.
   c. If the regression error u_i is conditionally heteroscedastic, the OLS estimators can still be consistent.
   d. If the conditional variance of u_i is heteroscedastic and its functional form is known and can be estimated exactly, we may prefer a weighted least square estimator to an OLS estimator because of the efficiency consideration.
   e. None of the above choices (a)-(d).

Please use the following information to answer Questions 9 and 10. Suppose you estimate the following model, with standard errors of the coefficients in parentheses.

$$\widehat{Price} = 200 + 50NBR + 50Lsize + 0.1\,Age$$
$$(9.76)\quad (0.021)\quad (0.0072)\quad (12.23) \qquad \bar{R}^2 = 0.55$$

9. Lot size ($Lsize$) is measured in square meters. If you change the unit of measurement to square centimeters. What will be the new estimated coefficient of $Lsize$?
   a. 50×10^{-4}
   b. 50×10^{-2}
   c. 50×10^{2}
   d. 50×10^{4}
   e. None of the above choices (a)-(d).

10. H_0: The coefficients of *NBR* and *Age* are jointly zero. H_a: The coefficient of *NBR* and *Age* are not jointly zero. You H_0 using a F-test, with the test statistic = 2.2. Probability(F-statistic > 2.2) = 0.03. Which of the following is correct?
    a. You can reject H_0 at the 10% level, but not at the 5% level.
    b. You can reject H_0 at both the 10% and 5% levels, but not at the 1% level.
    c. You can reject H_0 at the 10%, 5% level, and 1% levels.
    d. You cannot reject H_0 at the 10%, 5% level, or 1% level.
    e. None of the above choices (a)-(d).

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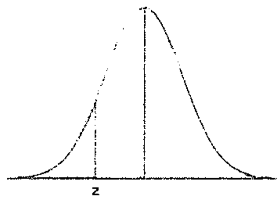
Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

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Part II: Fill-in-the-blank questions ※ 本大題請於試卷內之「非選擇題作答區」標明題號依序作答。

Note:

(1) The answers that are not integer values are rounded to two decimal places.

(2) Write down your answers on the answer sheet.

(3) Each of Questions 1 to 7 is worth 6 points and Question 8 is worth 8 points.

1. Suppose that X and Y are two continuous random variables with the joint probability density function $f(x, y) = x + 2y$ for $0 < x < 2$ and $0 < y < 2$. The covariance of X and Y is __________.

2. A random variable X has a mean $\mu = 5$ and variance $\sigma^2 = 36$. Based on Chebyshev's inequality, a lower bound on $\Pr(-7 < X < 17)$ is __________.

3. Suppose that X is a father's height, Y is a mother's height, Z is a child's ultimate height (as an adult), and G is defined as: $G = 1$ if the child is a boy and $G = -1$ if the child is a girl. Let $M = (X + Y)/2$. Suppose that, conditional on known values of G, X, and Y, Z has a continuous uniform distribution with range $[M - 5 + 3G, M + 5 + 3G]$. If a girl's father is 4 inches taller than her mother, the probability that she will grow to be taller than her mother is __________.

4. A manufacturer of car tires claims that his tires will last, on the average, 4 years with a standard deviation of 1 year. Given that 5 of these tires have lifetimes of 2.1, 2.2, 3.0, 3.6, 4.1 years, if someone wants to test whether his tires have a standard deviation of 1 year, the figure of the appropriate test statistic is equal to __________.

5. You are going to carry out an experiment to determine whether surface finish has an effect on the endurance limit of steel. It is expected that polishing increases the average endurance limit. In such an experiment, you want to detect that polishing fails to have an effect with a probability of 0.99 (i.e., $\alpha = 0.01$) and you also want to detect a change in the average endurance limit of 700 units by a probability of at least 0.9. If it is known that the standard deviation of the endurance limit of the steel is 420 units, then the sample size that would be needed to perform this experiment is __________. Assume that for $\alpha = 0.01$, $z_\alpha = 2.5$ and for $\beta = 0.10$, $z_\beta = 1.5$.

6. How large a sample must be taken in order that you are 95 percent certain that $\bar{X}_n$ is within 0.5 of the standard deviation of the mean? __________.

7. Ten trials are conducted of a given system with the following results: S, S, F, S, S, S, F, S, S, S ("S" represents success and "F" represents failure), assuming that the trials are independent of each other. The maximum likelihood estimate of the probability of successful operation is __________.

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8. At the beginning of the school year, the mean score of a group of 49 sixth-grade students on an achievement test in reading was 70 with an unbiased sample variance of 36. At the end of the school year, the mean score on an equivalent form of the same test was 74 with an unbiased sample variance of 25. The correlation between scores made on the initial and final testing was 0.6. To test whether the mean scores on these two exams are significantly different based on the Student *t*-test, the figure of the test statistic is equal to __________.

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