

**Part I. Multiple-choice questions**

Note:

- (i) Please choose only one of the answer choices (a)-(e).
- (ii) In the following questions,  $N(a, b)$  refers to a normal distribution with the mean equal to  $a$  and variance equal to  $b$ .
- (iii) Write down your answers on scantron answer sheet.
- (iv) Each question is worth 5 points.
- (v) The table of standard normal cumulative probability is used for answering the relevant questions in both Parts I and II.

Please use the following information to answer Questions 1 and 2. Suppose you estimate the following model, with standard errors of the coefficients in parentheses.

$$\widehat{Price} = 2.3 + 0.4 \text{ Size} + 0.5 \text{ BM} + 0.1 \text{ Age}$$

$$(3.56) (0.0021) \quad (0.212) (9.23) \quad \bar{R}^2 = 0.05$$

1. Firm size (*Size*) is measured in million dollars. If you change the unit of measurement to billion dollars. What will be the new estimated coefficient of *Size*?
  - a.  $0.4 \times 10^{-3}$
  - b.  $0.4 \times 10^{-2}$
  - c.  $0.4 \times 10^2$
  - d.  $0.4 \times 10^3$
  - e. None of the above choices (a)-(d).
2.  $H_0$ : The coefficient of *BM* is zero.  $H_a$ : The coefficient of *BM* is not zero. You test  $H_0$  using a t-test, with the test statistic = 2.36.  $\text{Probability}(t\text{-statistic} > 2.36) = 0.0091$ . Which of the following is correct?
  - a. You can reject  $H_0$  at the 10% level, but not at the 5% level.
  - b. You can reject  $H_0$  at both the 10% and 5% levels, but not at the 1% level.
  - c. You can reject  $H_0$  at the 10%, 5% level, and 1% levels.
  - d. You cannot reject  $H_0$  at the 10%, 5% level, or 1% level.
  - e. None of the above choices (a)-(d).
3. If  $X_1, X_2, \dots, X_n$  are mutually independent normal random variables, with their means all equal to 3 and variances equal to 10, what is the variance of  $Y = \sum_{i=1}^n 2X_i$ ?
  - a. 400
  - b. 200
  - c.  $400n$
  - d.  $200n$
  - e. None of the above choices (a)-(d).
4.  $Y$  is distributed  $\chi_{10}^2$ , and  $K$  is distributed  $F_{10, \infty}$ . Suppose  $\text{Probability}(Y \leq 16) = 0.90$ . Please calculate  $\text{Probability}(K \leq 1.6)$ .
  - a. 0.09
  - b. 0.45

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- c. 0.75  
d. 0.90  
e. None of the above choices (a)-(d).
5. A random draw (with replacement) of 1000 balls from a pool of several million balls is conducted. Each ball is either blue or red. Let  $p$  denote the fraction of blue balls in the population. Let  $\hat{p}$  denote the fraction of blue balls in the sample. You want to test the competing hypotheses  $H_0: p = 0.4$  vs.  $H_1: p \neq 0.4$ . Suppose that you decide to reject  $H_0$  if  $|\hat{p} - 0.4| > 0.01$ . Note that the sample of 1000 balls is considered as a large sample. What is the type I error of this test?  
a. 0.178  
b. 0.256  
c. 0.367  
d. 0.488  
e. 0.519
6. Suppose you estimate a sample OLS regression model. The resulted regression equation is:  $Y_i = 4.1 + 3.5 X_i + \varepsilon_i$ .  $Y_i$ ,  $X_i$ , and  $\varepsilon_i$  denote the dependent variable, independent variable, and the regression residual, respectively.  $i$  indexes each individual observation. If  $X_i$  is equal to 2, what is the predicted value of  $Y_i$ ?  
a. 23.5  
b. 19.6  
c. 15.2  
d. 11.1  
e. None of the above choices (a)-(d).
7. Suppose a population regression model is:  $Y_i = \beta_0 + \beta_1 X_i + u_i$ .  $Y_i$ ,  $X_i$ , and  $u_i$  denote the dependent variable, independent variable, and the error term, respectively.  $i$  indexes each individual observation.  $\hat{\beta}_0$  and  $\hat{\beta}_1$  are the OLS estimators, and  $\hat{u}_i$  is the regression residual, using a sample of  $n$  observations. Which of the following statement is false?  
a.  $\sum_1^n \hat{u}_i = 0$ .  
b.  $\sum_1^n \hat{u}_i X_i = 0$ .  
c. Suppose  $E(u_i | X_i) \neq 0$  and the correlation between  $X_i$  and  $u_i$  is zero. Then, the OLS estimator  $\hat{\beta}_1$  is inconsistent.  
d. The assumption that the correlation between  $X_i$  and  $u_i$  is zero cannot be tested empirically.  
e. None of the above choices (a)-(d).

Please answer questions 8 and 9 using the following information:

$$\text{Equation 1: } \ln(\widehat{\text{Earnings}}) = 3.21 + 0.55 \text{ CollegeDegree.}$$

$$\text{Equation 2: } \ln(\widehat{\text{Earnings}}) = 3.86 + 0.65 \text{ CollegeDegree} + 0.25 \ln(\text{Total\_Assets})$$

$$n = 30,000, \bar{R}^2 = 0.491.$$

*Earnings* measures a worker's monthly earnings. *CollegeDegree* is an indicator variable that is equal to 1 for workers who have a college degree, and 0 otherwise. *Total\_Assets* refers to the total assets of the firm that employs the worker. We take natural logarithm on both *Earnings* and *Total\_Assets*. *Earnings* are measured in thousand dollars, and *Total\_Assets* are measured in million dollars.

8. In Equation 2, holding all other factors constant, when *Total\_Assets* increases by 1%, what is the predicted change in Earnings?
- Increase by 250 dollars.
  - Increase by 1284 dollars.
  - Increase by 0.25%.
  - Increase by 25%.
  - None of the above choices (a)-(d).
9. Suppose Equation 2 is the correct model. That is, a firm's total asset size is a determinant of an employee's earnings. Are larger firms (measured by total asset size) more likely than smaller firms to employ college graduates?
- Larger firms are more likely to hire college graduates.
  - Smaller firms are more likely to hire college graduates.
  - All sizes of firms are equally likely to hire college graduates.
  - We don't have sufficient information to answer this question.
  - None of the above choices (a)-(d).
10. Which of the following is false?
- If all Gauss-Markov conditions are satisfied, the OLS estimators are unbiased.
  - If all Gauss-Markov conditions are satisfied, the OLS estimators are consistent.
  - If all Gauss-Markov conditions are satisfied, the OLS estimators have the lowest conditional variance among all linear conditionally unbiased estimators.
  - If the conditional variance of  $u_i$  is heteroscedastic, then Gauss-Markov conditions are violated and the OLS estimators are biased.
  - None of the above choices (a)-(d).

## Part II: Fill-in-the-blank questions

### Note:

- The answers that are not integer values are rounded to two decimal places.
- Write down your answers on the answer sheet.
- Each of Questions 1 to 7 is worth 6 points and Question 8 is worth 8 points.

1. A recent report claims that college non-graduates get married at an earlier age than college graduates. To support the claim, random samples of size 100 were selected from each group, and the mean age at the time of marriage was recorded. The mean and standard deviation of the college non-graduates were 21.6 years and 3 years, respectively, while the mean and standard deviation of the college graduates were 23.2 years and 4 years, respectively. To test the claims of the report at the 0.05 level of significance, the figure of the appropriate test statistic is \_\_\_\_\_.

2. Let the joint probability function of  $X$  and  $Y$  be given by

$$p(x, y) = \begin{cases} \frac{1}{6}(x+y) & \text{if } x = 0, 1, 2 \text{ and } y = 1, 2 \\ 0 & \text{otherwise.} \end{cases}$$

$$P(X = 1 | Y = 2) = \underline{\hspace{2cm}}.$$

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3. Customers enter a restaurant at random at a rate of four per minute. Assume that the number entering this restaurant in any given time interval has a Poisson distribution. The probability that at least two customers enter the restaurant in a given 0.25-minute interval is \_\_\_\_\_.
4. A random variable  $X$  represents the number of defective mobile phone in a shipment of four iPhones to a local mobile phone store. Assume that each mobile phone is equally likely to be defective or non-defective, and also assume that each mobile phone is defective or non-defective independently of the other mobile phones. Then the expected number of defective mobile phones is \_\_\_\_\_.
5. Suppose that a firm's sales region has been divided into five territories, each of which was judged to have an equal sales potential. The actual sales volume for several sampled days is indicated below.

	Territory				
	A	B	C	D	E
Actual Sales	110	90	80	100	120

If the manager of the firm wants to test the significance of the difference between the observed and expected levels of sales, the figure of the appropriate test statistic is \_\_\_\_\_.

6. The time elapsed, in minutes, between the placement of an order of pizza and its delivery is random with the density function

$$f(x) = \begin{cases} 1/20 & \text{if } 20 < x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that it takes 15 minutes for the pizza shop to bake pizza once the shop receives the order. The mean of the time it takes for the delivery person to deliver pizza is \_\_\_\_\_.

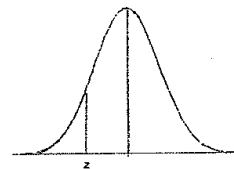
7. The following time series represents sales of imported cars in a city for the years 2011 through 2017:

Year	Sales of Imported Cars
2011	524
2012	549
2013	910
2014	1024
2015	1610
2016	1857
2017	2015

Suppose we want to estimate annual sales of imported cars in the year 2018. The expected number of imported car sales is \_\_\_\_\_.

8. A manufacturer produces a special alloy steel with an average tensile strength of 26,000 psi. A change in the composition of the alloy is said to increase the breaking strength but not affect the standard deviation which is known to be 400 psi. The manufacturer wants to conclude that the tensile strength has increased only if he is 99% sure of this ( $\alpha = 0.01$ ). If the average tensile strength is increased by as much as 300 psi, the manufacturer wants to err by not detecting the change at most 10% of the time ( $\beta = 0.10$ ). Assume that the  $z$  statistic with respect to the type I error is 2.33 and the  $z$  statistic with respect to the type II error is -1.28. The required sample size to meet these conditions is \_\_\_\_\_.

Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641