

第一部分：財務管理

1. (15 points)

Capital structure overlooks that firms have different capital requirements across their life cycle. In addition, the finance literature highlights various theoretical models that explain how the firms' life stages influence capital structure decisions, such as the Pecking Order, Trade-Off, and Agency Cost Theories. We like to know how firms in highly innovative industries have different capital requirements over their life cycle.

- (1) (5 points) In the beginning of the life cycle, firms are more likely to raise funds through issuing equity or using borrowing, why?
- (2) (5 points) When firms are matured, they are more likely to use debt or equity financing, why?
- (3) (5 points) Tony Lin is a founder of a highly innovative company. The company is facing a high-growth period. Tony insists to use debt financing to support the firm's new investment. Does Tony make the right decision? Why, or why not? Why does Tony make such a decision? Is there anything related to his personal preference?

2. (15 points)

To construct a portfolio using the mean-variance framework, one needs estimates of the expected return  $r_i$ , the variance  $\sigma_i^2$  and the co-variance  $\sigma_{ij}$  for each stocks  $i, j$ . For  $n$  stocks, you have to estimate a total of  $n(n-1)/2$  correlation coefficients. Single index models are used, to reduce this huge amount of needed estimates. Now we are going to construct a portfolio with two assets using the single index model.

- (1) (3 points) Please calculate the mean and variance of the stock index return and mean return for asset B.

Obs.	1	2	3	4	5	6	7	8	9	10	E(r)	Var(r)
Asset A	5.86	7.71	0.63	0.51	-0.39	-5.69	5.54	-5.58	17.04	-17.48	0.815	86.45185
Asset B	2.58	2.02	3.78	-3.16	7.68	3.34	0	7.8	6.54	-9.98		29.5472
Stock index	4.23	9.37	0.24	-0.61	-0.58	8.97	6.2	-0.46	13.22	-14.03		

We run the following regression for each asset.

$$r_i = \alpha_i + \beta_i r_M + e_i$$

- (2) (2 points) Please find out the estimates of  $\alpha_i$  and  $\beta_i$  for Asset A

Asset	$\alpha_i$	$\beta_i$	$\sigma_{ei}^2$
A			30.12613
B	0.918841	0.429815	21.078

- (3) (4 points) A Single Index Model (SIM) specifies two sources of uncertainty for a security's

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return: Systematic risk and firm-specific risk

Return variance	=	Systematic risk	+	Firm-specific risk
$\sigma_A^2$	=		+	
$\sigma_B^2$	=		+	

(4) (6 points) Now assuming a portfolio with weights for asset A and asset B are 0.7 and 0.3.

Please find out  $E(r_p)$  and  $\sigma_p^2$ .

Mean Return	=	$\alpha_p$	+	$\beta_i E(r_M)$
$E(r_p)$	=		+	

Return variance	=	Systematic risk	+	Firm-specific risk
$\sigma_p^2$	=		+	

3. (10 points)

The equity in a firm is a residual claim, i.e., equity holders lay claim to all cashflows left over after other financial claim-holders (debt, preferred stock etc.) have been satisfied. If a firm is liquidated, the same principle applies, with equity investors receiving whatever is left over in the firm after all outstanding debts and other financial claims are paid off. The principle of limited liability, however, protects equity investors in publicly traded firms if the value of the firm is less than the value of the outstanding debt, and they cannot lose more than their investment in the firm.

(1) (2 points) Beside the residual claim right mentioned above, what else rights that an equity investor hold?

(2) (1.5 point) The payoff to equity investors, on liquidation, can therefore be written as: Payoff

to equity on liquidation=		if $V > D$
=		if $V \leq D$

where,  $V$  = Value of the firm

$D$  = Face Value of the outstanding debt and other external

Please draw a figure of the payoff to equity investors (Y-axis) and the firm value (X-axis).

(3) (1.5 point) The payoff to debt investors, on liquidation, can therefore be written as: Payoff

to debt on liquidation=		if $V > D$
=		if $V \leq D$

where,  $V$  = Value of the firm

$D$  = Face Value of the outstanding debt and other external

Please draw a figure of the payoff to debt investors (Y-axis) and the firm value (X-axis).

接次頁

(4) The relationship between equity/debt and firm value can be described by the following assets, call option, put option, and pure bond. Please fill in the correct position (long, short, or NO) for an investor to take to describe the relation between equity/debt and firm value. (3 points)

Assets	Equity	Debt
Call option		
Put option		
Pure bond		

Please explain four reasons. (2 points)

4. (10 points, each question has 2 points)

Please write down the right choice for each question.

(1) Vincent Swift is a risk neutral investor, he will choose (A)A (B)B (C)C (D)D

Asset	E(r)	Var(r)
A	0.8	0.03
B	0.2	0.7
C	0.5	0.5
D	0.92	0.103

(2) Here are cash flow forecasts for two mutually exclusive projects.

	Year	0	1	2	3	4
cash flows Dollars	Project A	-120	33	51	70	58
	Project B	-100	60	58	52	2

Based on NPV, which project would you choose if the opportunity cost of capital is 12%? (A) A, (B) B, (C)the same, (D)cannot be decided.

(3) Based on M&M model, A firm has a debt-to-equity ratio of 1.0. If it had no debt, its cost of equity would be 15%. Its cost of debt is 10%. What is its cost of equity if there are no taxes or other imperfections? (A)21%. (B)18%. (C)15%. (D)20%.

(4) Both call and put options are impacted by the ex-dividend date. Put options become more and Call options become \_\_\_\_\_ due to the dividend payment. (A) expensive, expensive (B)cheaper, cheaper (C)expensive, cheaper (D)cheaper, expensive.

(5) A company (NTU Ltd.) that has paid a dividend of \$4 this year. Assuming a higher growth for the next 3 years at 12% and stable growth of 6% thereafter, let's calculate the value of the stock using a two-stage dividend discount model. The required rate of return is 10%. (A)4.48 (B)5.95 (C)124.33 (D)154.54

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第二部分：財金數學

1. (5 points) Consider the system of following questions

$$2x_1 + x_2 + x_3 = 2$$

$$-x_1 + 2x_2 = 1$$

$$x_1 - x_2 + 2x_3 = -2$$

where  $x_1, x_2, x_3$  are unknown. Please use the matrix-vector product to express the  $3 \times 3$  linear system

2. (5 points) Following previous question, please find reduced row echelon form (rref)

3. (5 points) Please find a matrix C solution so that  $(BC)^T - A = 0$ , where

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ -1 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$$

4. Consider a two-period economy ( $t = 0, 1$ ). The operating profit of firm  $j$  can be expressed as  $\pi_{jt} = e^{x_t} A_t^\alpha k_{jt}$ , where  $e^{x_t}$  is the exogenous profitability shock;  $A_t$  is a technology shock at time  $t$ ;  $k_{jt}$  is private capital of firm  $j$ .  $\alpha > 0$  is assumed. The depreciation rate of capital  $k$  for firm  $j$  is  $\delta_j$ . Thus, the investment of firm  $j$  at  $t = 0$  is  $i_{j0} = k_{j1} - (1 - \delta_j)k_{j0}$ . At  $t = 1$ , firm  $j$  has a liquidation value of  $(1 - \delta_j)k_{j1}$ . The investment in private capital involves adjustment cost  $c(k_{j0}, i_{j0}) = \frac{c}{2} \left( \frac{i_{j0}}{k_{j0}} \right)^2 k_{j0}$ . The stochastic discount factor between time 0 and 1 is  $m$ , where  $\log(m) = \log\beta + \gamma(x_0 - x_1)$ , where  $0 < \beta < 1$ , and  $\gamma > 0$ . Firm  $j$  then chooses  $i_{j0}$  to maximize its value, which is the sum of the discounted cash flows from the two periods:

$$\max_{\{i_{j,0}\}} e^{x_0} A_0^\alpha k_{j0} - i_{j0} - \frac{c}{2} \left( \frac{i_{j0}}{k_{j0}} \right)^2 k_{j0} + E_0 [m(e^{x_1} A_1^\alpha k_{j1} + (1 - \delta_j)k_{j1})].$$

$E[\cdot]$  is an expectation operator. From standard asset pricing equation,  $E_0[r_1 - r_f] \approx -\text{Cov}_0(r_1, m)$ . Specifically, the cash flows from  $t = 0$  are equal to the operating profit minus the investment cost and the adjustment cost. The cash flows from  $t = 1$  are equal to the operating profit plus the liquidation value.

- (1) (10 points) Please drive FOC for maximum firm value, and express it as a form of marginal cost equal to marginal revenue.
- (2) (10 points) Note that the investment return,  $r_1'$ , defined as the ratio of marginal benefit to marginal cost of investment, is equal to the firm's stock return ( $r_1$ ) under equilibrium. Please show that  $E_0[mr_1'] = 1$
- (3) (10 points) Please derive expected excess return:  $E_0[r_1 - r_f]$ , which is a function of the covariance between the operating profit at  $t = 1$  of firm  $j$  and the aggregate productivity shock.
- (4) (5 points) Show that the expected excess return of firm  $j$  is positively associated with technology shock at time 1.