※ 注意：請於試卷內之「非選擇題作答區」作答，並應註明作答之題號。

1. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

   a) Find the eigenvalues of A. (5%)
   b) Find a bases for each eigenspace. (5%)
   c) Find an orthogonal matrix Q and diagonal matrix Λ such that $Q^T A Q = \Lambda$. (10%)

2. Let $A = \begin{bmatrix} C & 1 & 1 \\ -1 & C & 1 \\ 1 & 1 & C \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 2C \\ 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. For what values of C does the equation AX = B have

   a) One solution. (7%)
   b) No solution. (7%)
   c) Infinitely many solutions. (6%)

3. Let $A = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 5 & 0 & -1 \\ 2 & 0 & 3 & 4 \\ 0 & -1 & 1 & 0 \end{bmatrix}$. Compute the following determinants:

   a) $\det(A)$. (4%)
   b) $\det(2A^{-1})$. (4%)
   c) $\det(A^T A)$. (4%)
   d) $\det(A^T - A)$. (8%)

4. Suppose A and B are n × n matrices. A is symmetric and B is skew symmetric. Determine if AB is symmetric, skew symmetric, both, or neither. Either give a proof, or find an example. (10%)

5. Let V be a 2-dimensional subspace of $\mathbb{R}^n$ with basis $T = \{v_1, v_2\}$. Suppose that $\|v_1\| = \sqrt{3}$, $\|v_2\| = \sqrt{5}$, and $v_1 \cdot v_2 = 2$. Let $\mathbf{u} = v_1 - 2v_2$. Find $\|\mathbf{u}\|$. (10%)

6. Let $L\colon \mathbb{R}^2 \to \mathbb{R}$ be the function $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \sqrt{x^2 + y^2}$. Is L a linear transformation? Why or why not? (10%)

7. Let $S = \left\{\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}\right\}$. Let c be a constant. For what values of C is the matrix $\begin{bmatrix} c^2 & -5c \\ c^2 - 4 & -6 \end{bmatrix}$ in the span of S? (10%)

試題隨卷繳回