1. (10 points) Let $X \sim Beta(\alpha,\beta)$ and $Y \sim Beta(\alpha+\beta,\gamma)$ be two independent variables of beta distribution. Find the distribution of XY by making the transformations U=XY, V=Y. The pdf of beta distribution, Beta(a,b), is given by
$$f(x|a,b)=\frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)}\,x^{a-1}(1-x)^{b-1},\qquad 0<x<1.$$

2. Let $X_1,\dots,X_n$ be a random sample from the following density function
$$f(x|\theta)=\frac{2}{\sqrt{\pi\,\theta}}\,exp\left\{-\frac{x^2}{\theta}\right\},\qquad x>0,\qquad \theta>0.$$
(a) (5 points) Find the MLE (maximum likelihood estimator) of θ. Is it an unbiased estimator?
(b) (10 points) Use the MLE in (a) to obtain a $100\times(1-\alpha)\%$ confidence interval for θ.

Hint: $\frac{2X_i^2}{\theta}$ is distributed as a Chi-square distribution.

3. Let $X_1,\dots,X_n$ be a random sample from the distribution with pdf $f(x|\theta)=\frac{\theta x^{\theta-1}}{3^\theta}$, $0<x<3$.
(a) (5 points) Find the method of moment estimator of θ.
(b) (10 points) Find the limiting distribution of $\sqrt{n}(T-\theta)$ as $n\to\infty$ by the Delta method, where T is the method of moment estimator of θ.

Hint: the Delta method is stated as the following
If $\sqrt{n}(W-\theta)\to N(0,\sigma^2)$, $as\ n\to\infty$; then under regularity conditions
$$\sqrt{n}\big(g(W)-g(\theta)\big)\to N\left(0,\sigma^2\big(g'(\theta)\big)^2\right).$$

4. Suppose that $X_1,\dots,X_n$ are independent identically distributed random variables from the normal distribution with unknown mean μ and known variance σ^2. Consider the parameter function $g(\mu)=e^{2\mu}$.
(a) (5 points) Find the uniform minimum variance unbiased estimator (UMVUE) of $g(\mu)$.
(b) (5 points) Find the Cramér-Rao lower bound (CRLB) for the variance of unbiased estimator of $g(\mu)$. Is the CRLB attained by the variance of the UMVUE?

5. Let X be the number of calls received during any one hour, and follow a Poisson distribution with pmf: $P(X=x|\lambda)=\frac{\lambda^x}{x!}e^{-\lambda}, x=0,1,2,\dots$

To test $H_0\colon\lambda=4\ \ vs.\ \ H_A\colon\lambda=1$ and we know

	$x=0$	$x=1$	$x=2$	$x=3$	$x=4$
$P(X=x)=\frac{4^x}{x!}e^{-4}$	0.02	0.07	0.15	0.20	0.20
$P(X=x)=\frac{1^x}{x!}e^{-1}$	0.37	0.37	0.18	0.06	0.02

(a) (2 points) When significant level is set as 0.05, fine the rejection region of X.
(b) (2 points) When $X\in\{0,1\}$, reject H_0. Find the probability of type I error.
(c) (2 points) When $X\in\{0,1\}$, reject H_0. Find the probability of type II error.
(d) (2 points) When $X\in\{0,1\}$, reject H_0. Find the power of the test.
(e) (2 points) When hypothesis test: $H_0\colon\lambda\geq 4\ \ vs.\ H_A\colon\lambda<4$ with the same significant level α given in Question(a), find the rejection region.
(f) (2 points) When hypothesis test: $H_0\colon\lambda\geq 4\ \ vs.\ H_A\colon\lambda<4$ with the same significant level α given in Question(a), find the infimum (inf) of testing power.

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6. Student A use simple linear regression to test the linear relationship between Y and X. The summary of result shown as below:

Call: lm(formula = Y ~ X)
Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.1049	1.6875	0.062	0.951
X	1.8430	0.3157	5.838	7.12×10^{-6}

Residual standard error: 3.862 on 22 degrees of freedom

Please fill the blanks of ANOVA table based on the information of above result.

Analysis of Variance Table
Response: Y

	Df	Sum Sq	F value	Pr(>F)
X	(A)	508.38	(C)	(D)
Residuals.	(B)	328.15		

(a) (2 points) (A) =?
(b) (2 points) (B) =?
(c) (2 points) (C) =?
(d) (2 points) (D) =?
(e) (2 points) find R^2 of this regression analysis.

Y1 and X1 are separately the normalized Y and X. Using simple linear regression to test the linear relationship between Y1 and X1. The summary of result was shown as below, and answer following questions.

Call: lm(formula = Y1 ~ X1)
Coefficients:

	Estimate	Std. Error.	t value	Pr(>|t|)
(Intercept)	1.913×10^{-16}	0.1307	0.000	1
X1	0.7796	0.1335	(E)	(F)

(f) (2 points) (E) =?
(g) (2 points) (F) =?
(h) (2 points) Find the sample correlation coefficient of X and Y.
(i) (2 points) Find the F statistic of this linear regression analysis.

7.
(a) (4 points) Please state the Neyman-Pearson Theorem.
(b) (6 points) Please prove the Neyman-Pearson Theorem.

8. Let X_1, X_2 be a random sample from the distribution having pdf $f(x|\beta) = \beta e^{-\beta x},\ 0 < x < \infty$.

When $\frac{\prod_{i=1}^{2} f(x_i|\frac{1}{2})}{\prod_{i=1}^{2} f(x_i|1)} \le \frac{1}{2}$, we reject H_0: $\beta = \frac{1}{2}$ and accept H_1: $\beta = 1$.

Hint: the pdf of $Gamma(\alpha, \beta)$ is given by $f(y|\alpha,\beta) = \frac{y^{\alpha-1}\beta^{\alpha}e^{-\beta y}}{\Gamma(\alpha)}, y > 0$

(a) (5 points) Find the significant level of this test.
(b) (5 points) Find the power of this test.

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