題號： 256

1. Curves of constant temperature T(x, y)=const in a temperature field are called isotherms. Their orthogonal trajectories are the curves along which heat will flow (in regions that are free of sources or sinks of heat and are filled with a homogeneous medium). If the isotherms are given by $y = c - \frac{1}{2}x^2$,what are the curves of heat flow? (15%)

2. Solve: $x^3y''' + x^2y'' - 2xy' + 2y = 0$ (10%)

3. Find the Maclaurin series of $f(z) = \tan^{-1} z$ (10%)

4. Let M_n is the n×n matrix with 3's on the main diagonal except the first entry which is 1, 1's directly above the main diagonal and (-1)'s directly below the main diagonal, and 0's elsewhere. For example,

$$M_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 3 & 1 & 0 \\ 0 & -1 & 3 & 1 \\ 0 & 0 & -1 & 3 \end{bmatrix}$$

We define $D_n = \det(M_n)$. Please (a) Find D_1 and D_2, (b) If $n \geq 3$, find a formula for D_n in terms of D_{n-1} and D_{n-2}, (c) Find a formula for D_n in terms of n. (15%)

5. Given $v_x \equiv v_x(y)$,

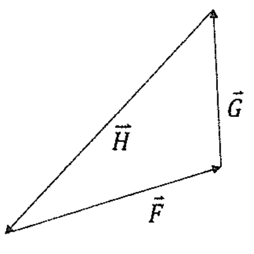
(a) solve $-\mu\left[\frac{d^2v_x}{dy^2}\right] + \rho v_0 \frac{dv_x}{dy} = \frac{P_0 - P_L}{L}$, if $v_x = 0$ at $y = 0$ and $y = B$;

(b) compute the value of y for which v_x is maximum;

(c) integrate $v_x(y)$ from $y = 0$ to B.

(20%)

6. $\vec{F}, \vec{G}, \vec{H}$ are shown in the following figure, please prove that $\vec{F} \times (\vec{G} \times \vec{H}) = \vec{G} \times (\vec{H} \times \vec{F}) = \vec{H} \times (\vec{F} \times \vec{G}) = 0$. (10%)



7. Show that if $u \equiv u(x, y)$ and $\nabla^2 u = 0$ (Laplace's equation), then $v \equiv u(x^2 - y^2, 2xy)$ would also fulfill Laplace's equation. (10%)

8. Find the convolution of $f(t) = e^{-t}$ and $g(t) = \sin(t)$. (10%)

試題隨卷繳回