1. Let $A=\begin{bmatrix}2&3&3\\0&5&7\\6&9&9\end{bmatrix}$. (a) Factor *A* into *LU*. (b) Find a basis for the row space of *A*. (c) Find a basis for the column space of *A*. (20%)
2. True or false, with reason if true and counterexample if false: (15%)
   (a) If *A* and *B* are symmetric then *AB* is symmetric.
   (b) If *AB* = *B* then *A* = *I*.
   (c) All geometric progressions $(x_1, kx_1, k^2x_1, \ldots)$ allowing all k and x_1 are subspace of R^∞.
   (d) If subspace V is orthogonal to W, then $V^\perp$ is orthogonal to $W^\perp$.
   (e) If *A* is invertible and *B* is singular, then *A* + *B* is invertible.
3. $A^T=\begin{bmatrix}1&3&4&5&7\\-6&6&8&0&8\end{bmatrix}$ (20%)
   (a) Find an orthonormal basis for the column space of A.
   (b) Write A as QR, where Q has orthonormal columns and R is upper triangular.
   (c) Find the least-squares solution to Ax = b, if b = [-3, 7, 1, 0, 4]^T.
4. Solve $\frac{d\bar{u}}{dt}=A\bar{u}$ with $\bar{u}(0)=[1, 0, 1]^T$

   $A=\begin{bmatrix}1&2&1\\0&3&6\\0&0&4\end{bmatrix}$. (15%)
5. Find the 3 by 3 symmetric matrix A and its pivots, rank, eigenvalues, and determinant: $\bar{x}^T A\bar{x}=(x_1-x_2+2x_3)^2$ (20%)
6. Identify the curve and sketch the graph. $x^2+xy+y^2+3x+3y=3$. (10%)

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