

1. (15%) A magnetic field is given in the xz -plane by $\vec{B} = \sin(x - v_0 t) \vec{a}_y$, Wb/m². Consider a rigid square loop situated in the xz -plane with its vertices at $(x, 0, 1)$, $(x+1, 0, 1)$, $(x+1, 0, 2)$, and $(x, 0, 2)$. Find the emf induced around the loop in the sense defined by connecting the above points in succession.

2. (15%) Charge is distributed with uniform density ρ_0 C/m³ in the spherical region $a < r < 2a$. Find the electric displacement field \vec{D} everywhere.

3. (20%) A current distribution is given in cylindrical coordinates by

$$\vec{J} = \begin{cases} J_0 \vec{a}_z & \text{for } r < 3a \\ -J_0 \vec{a}_z & \text{for } 4a < r < 5a \end{cases}$$

- (a) Find the magnetic field everywhere.
 (b) Find the energy stored in the magnetic field of the current distribution per unit length in the z -direction.

4. (15%) Let us consider the charge distribution given by

$$\rho = \begin{cases} -\rho_0 & \text{for } -a < x < 0 \\ 0.5\rho_0 & \text{for } a < x < 3a \end{cases}$$

where ρ_0 is a constant. Find the electric displacement field \vec{D} everywhere.

5. (15%) For a uniform plane wave propagating in the $-z$ -direction in a material medium with $\sigma=0$, $\epsilon=4\epsilon_0$, $\mu=\mu_0$, the magnetic field intensity in the $z=0$ plane is given by

$$\vec{H}|_{z=0} = \cos(10^8 t) \vec{a}_y \text{ A/m}$$

- (a) Find the impedance of the medium for the field.
 (b) Find the phase velocity of the wave.
 (c) Find the associated electric field $\vec{E}(z,t)$.

6. (20%) Consider a structure with three regions, region 1, 2, and 3. Region 1, $x > d$, is free space. Region 2, $0 < x < d$, is a perfect dielectric of $\epsilon = 4\epsilon_0$ and $\mu = \mu_0$. Region 3, $x < 0$, is a perfect conductor. The electric and magnetic fields in the region 2 are given at a particular instant of time by

$$\vec{E} = E_1 \cos \pi x \sin 2\pi z \vec{a}_x$$

$$\vec{H} = H_1 \cos \pi x \sin 2\pi z \vec{a}_y$$

- (a) Find ρ_s and \vec{J}_s on the surface $x=0$.
 (b) Find \vec{E} for $x=d^+$, that is, immediately adjacent to the $x=d$ plane and on the free-space side, at that instant of time.