

1. (15%) Find the solutions of the following ODE's.

(a) $y' = y e^{3x}$

(b) $y'' + 3y' + 2.25y = -10 e^{-1.5x}$

(c) $x^3 y''' - 3x^2 y'' + 6xy' - 6y = x^4 \ln x$

2. (a) (7%) Find the Laplace transform for $y(t) = e^{-t} + \sinh(t) - t$

(b) (8%) Given a second order differential equation $y'' - y = t$ with initial conditions $y(0) = y'(0) = 1$, please find the solution by using Laplace transform method.

3. (10%) $\mathbf{F} = [x, y, z]$ is a three-dimensional vector. Please calculate the value of a surface integral $\iint_S [\mathbf{F} \cdot \mathbf{n}] dA$, where S is a surface defined as $x^2 + y^2 + z^2 = 9$ and \mathbf{n} is the normal vector of S .

4. (10%) Solve the following ODE

$$x(x-1)y'' + (3x-1)y' + y = 0$$

5. (15%) Solve the PDE below.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \text{ B.C. } \begin{cases} u = 0 & , \text{ at } x = 0 \\ u = 2 - y & , \text{ at } x = 2 \\ \frac{\partial u}{\partial y} = 0 & , \text{ at } y = 0 \\ \frac{\partial u}{\partial y} = 0 & , \text{ at } y = 2 \end{cases} \quad \text{for } 0 < x < 2, 0 < y < 2$$

6. (15%) Solve the PDE below.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{I.C. } u(x, t=0) = 0, \text{ B.C. } \begin{cases} u = t & , \text{ at } x = 0 \text{ for all } t > 0. \\ u = 3 & , \text{ at } x = 1 \text{ for all } t > 0. \end{cases}$$

7. (10%) Find the Fourier integral of $f(x)$.

$$f(x) = \begin{cases} \sin x, & \text{if } |x| < \pi \\ 0, & \text{if } |x| > \pi \end{cases}$$

8. (10%) Find the inverse Fourier transform of the function $\frac{6e^{i\omega} \cos(4\omega)}{9 + \omega^2}$.

試題隨卷繳回