1. (50%) Lymphangions (淋巴管節), functional units of a lymph vessel (淋巴管) that lie between two semilunar values (瓣膜), are muscular and capable of contracting themself one by one to propel the lymph (淋巴液) forward and increase the fluid pressure (figure 1). A lymph vessel now is modeled by a micro-tube with a deformable wall as shown in figure 2. The micro-tube has a fixed length L and a radius R_0 when the wall is undeformed. We assume that a lymph vessel is very thin, I.e., $R_0 \ll L$. We model the operations of lymphangions as a wall deformation in a form of traveling wave, i.e, $\Delta R = \Delta R(kz - \omega t)$, where z is the streamwise coordinate, k and ω are wavenumber and angular velocity of the traveling wave, whose traveling velocity is $c = \omega/k$. Thus, the deformable wall is described by an axisymmetric surface at $r = R(z,t) = R_0 + \Delta R(kz - \omega t)$. Consider the flow of lymph in the micro-channel is incompressible, axisymmetric and without swirling. The lymph is assumed to have constant density ρ and dynamics viscosity μ.

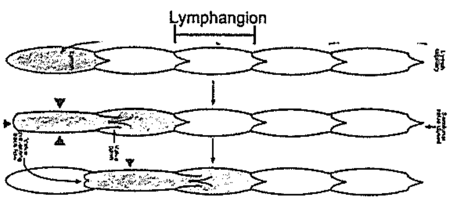

Figure 1. Propagation of lymph.

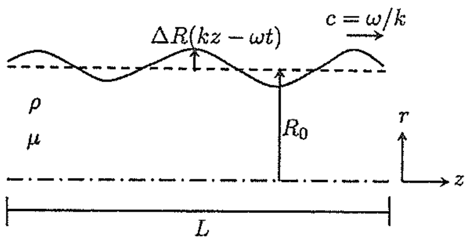

Figure 2. The schematics of the micro-tube.

The following incompressible axisymmetric no-swirling Navier-Stokes equations in the cylindrical coordinate will be helpful:

(C) $$\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{\partial u_z}{\partial z} = 0 ,$$

(M_r) $$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + u_z\frac{\partial u_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) - \frac{u_r}{r^2} + \frac{\partial^2 u_r}{\partial z^2}\right] ,$$

(M_z) $$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{\partial^2 u_z}{\partial z^2}\right] ,$$

where p is the pressure and u_r and u_z are the radial and streamwise components of velocity.

(1) (5%) Let the scales of the streamwise and radial velocities of the lymph flow be U and W, respectively. Let the streamwise and radial positions in the lymph flow be scaled by R_0 and L, respectively. From scale analysis of the continuity equation, what can you tell about the magnitudes of U and W?

(2) (5%) From scale analysis of the momentum equation in the z direction, under what condition can the momentum equation in the z direction be reduced to the equation of Poiseuille flow? What should the scale of pressure be?

(3) (5%) From scale analysis of the momentum equation in the r direction, what conclusion can be drawn about the pressure distribution with the results from part (b)?

(4) (5%) Consider a control volume in the micro-tube that has a cross-sectional area $A(z,t) = \pi R^2(z,t)$ and an infinitesimal length dz. From control volume analysis, show that $\frac{\partial Q}{\partial z} + \frac{\partial A}{\partial t} = 0$, where $Q(z,t) = \int_0^{R(z,t)} u_z(2\pi r)dr$ is the volume flow rate.

(5) (10%) In the case of part (b), write down the boundary conditions of the Poiseuille flow in the micro-tube and solve for $u_z(r,z,t)$. Compute $Q(z,t)$ by integrating $u_z(r,z,t)$ directly.

(6) (10%) Since $R(z,t)$ is in a form of a traveling wave, $A(z,t)$ and $Q(z,t)$ are too in a form of traveling wave. From the results in part (d) show that $Q(z,t) = cA(z,t) + Q_0(t)$. Together with the results from part (e), derive an expression of the pressure gradient in the z direction in terms of $Q_0(t)$, R_0, ΔR, c, and μ.

(7) (10%) Consider the deformation of the wall is small, i.e., $\Delta R \ll R_0$. Perform the Taylor expansion of the pressure gradient about $\Delta R = 0$ to the first order and show that it is possible to obtain a pressure rise when there is a traveling wave propagating on the deformable wall.

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2. (50%) A solid sphere of radius R and density ρ_s is immersed in the center of a deep tank of incompressible fluid of viscosity μ and density ρ. At time zero, the sphere is suddenly released and falls in a straight line under the action of gravity. The sphere velocity u(t) increases from zero and reaches a steadily falling terminal velocity U. The quasi-steady drag F_D acting on the sphere may be calculated by reading C_D-Re plot shown in Figure 3 where C_D is the *drag coefficient* and $Re=2R\rho U/\mu$ is the particle Reynolds number.

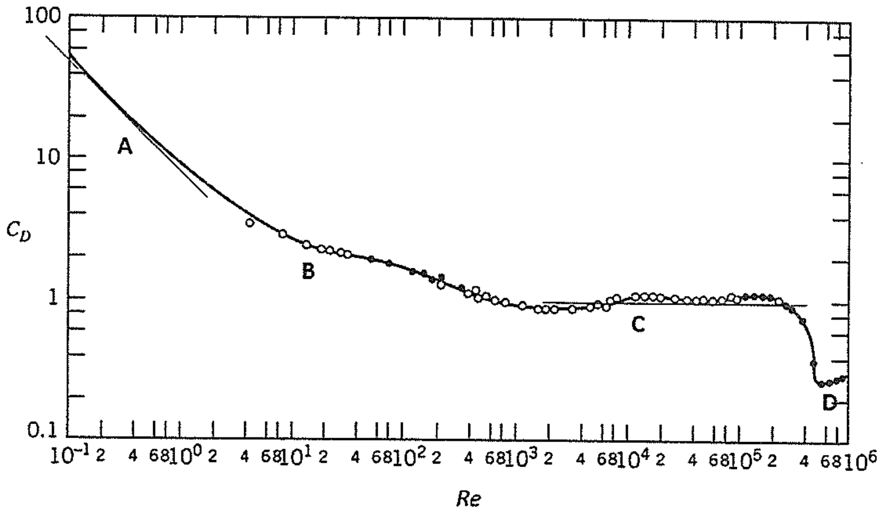

Figure 3

(1) (10%) Formulate a general dynamic equation to describe the sphere transient unsteady motion u(t) from time zero with a proper initial condition. Explain the physical meaning of every term in your equation.

(2) (5%) If the sphere motion falls in the low-Re flow regime (Regime A), C, the drag coefficient can be described by $C_D=24/Re$. Determine the sphere terminal velocity U.

(3) (7%) Based on the Navier-Stokes equation, use scaling arguments to explain why the total drag in the low-Re regime (Regime A) always follows the same relation that $\log C_D \sim -\log(Re)$, or equivalently, $F_D \sim \mu RU$.

(4) (8%) Perform dimension analysis to find all the relevant non-dimensional flow variables with one that involves the transient timescale T as a dimensionless time scale, T^*.

(5) (5%) Now, we consider the motion of a spherical liquid droplet of identical radius R, density ρ_L ($\rho<\rho_L<\rho_s$) and viscosity μ_L that also falls at its terminal velocity U_L in the low-Re regime. Do you expect a greater or a lower C_D than that of the solid sphere if the liquid droplet falls at the same speed as the solid sphere? Why?

(6) (5%) Compare the terminal velocity of the spherical liquid droplet and the solid sphere. Under what condition they may fall at the same speed?

(7) (5%) If the sphere now moves in Regime B, the experienced C_D decays with Re at a slower rate than the decay rate in Regime A. Can you explain it use the flow process?

(8) (5%) Use flow process to explain why C_D remains nearly constant in Regime C. What happens when there is a sudden drop of C_D in Regimes D and explain its flow process.

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