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國立臺灣大學 110 學年度碩士班招生考試試題

科目：工程數學(A)

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1. For a matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

- (a) (5%) What is the rank of A?
- (b) (5%) Calculate A^{-1}
- (c) (15%) Diagonalize A
- (d) (5%) Calculate A^{10}

2. (20%) Solve

$$(6x^6 - x^2y) + (-x^3 + x^2y^2)y' = 0, \text{ with } y(0) = 3$$

3. (20%) Solve

$$y'' + 4y = r(t)$$

$$r(t) = |t|, \quad -\pi \leq t \leq \pi, \quad r(t) = r(t + 2\pi)$$

4. Consider the heat equation with a negative source:

$$u_t = u_{xx} - 6x, \quad 0 < x < 1, \quad t > 0$$

$$u(0, t) = u(1, t) = 0$$

$$u(x, 0) = u_0, \quad 0 < x < 1$$

where $u_0 > 0$ is a constant

(a) (5%) Find the steady-state solution u_{eq}

(b) (25%) Solve $u(x, t)$

(hint: you may want to transform the given heat problem by $v(x, t) = u(x, t) - u_{eq}$)

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