※ 注意：請於試卷內之「非選擇題作答區」依序作答，並應註明作答之大題及小題題號。

1 Solow growth model (25 points)

Consider a Solow growth model with exogenous technological growth but no population growth. Let N denote the number of fixed labor supply in each period. Suppose that output is produced according to the constant return to scale production function, $Y_t = F(K_t, Z_t N) = K_t^{\alpha}(Z_t N)^{1-\alpha}$ where Z_t, and K_t denote respectively the labor-augmenting productivity, and capital in period t. In addition, Z_t evolves according to $Z_t = Z_{t-1}\gamma_Z$. The law of motion for capital stock is $K_{t+1} = (1-\delta)K_t + I_t$. The aggregate consumption C_t equals a constant fraction, $1-s$, of aggregate production, Y_t, i.e., $C_t = (1-s)Y_t$ and the aggregate investment equals the remaining fraction s of aggregate production, i.e., $I_t = sY_t$.

1 (5 points) Define y_t as output per effective worker, i.e., $y_t = \frac{Y_t}{Z_t N}$ and k as capital per effective worker, i.e., $k_t = \frac{K_t}{Z_t N}$. Use the constant return to scale production function to express the relationship between y and k.

2 (5 points) Derive the law of motion for capital per effective worker, express the relationship in terms of saving rate and depreciation rate and growth rate of labor-augmenting productivity. Compute the steady-state quantity of capital per effective worker and output per effective worker in terms of s, δ, and γ_z.

3 (5 points) Use a diagram to show the steady state quantity of capital per effective worker.

4 (5 points) What are the long-run growth rate of output per effective worker, and capital per effective worker? What are the long-run growth rate of output per worker, and capital per worker?

5 (5 points) Use a diagram to show the effect of an increase in saving rate, s, in the Solow Model. In particular, please show both the original and new steady state quantity of capital per worker in the diagram, and explain dynamic adjustment of capital per worker from the initial steady state to the new steady state.

2 The labor search model (25 points)

Assume the workers in the model are all in the labor force. That is, the workers are either employed or unemployed, with U denoting the fractions of workers who are unemployed, and $(1-U)$ the fraction who are employed. The jobs of the employed differ according to the wage that they are paid. Each period, there is a probability p that the unemployed worker can draw one job offer from the wage distribution, $H(\hat{w}) = Prob(w \geq \hat{w})$. The worker has the option to reject the offer, in which case, he or she receives unemployment compensation. Alternatively, the worker can accept the offer to work at w, in which case, he or she receives w each period. Let $V_e(w)$ denote the expected value for a worker who has offer with wage w in hand. $V_e(w)$ is an increasing and concave function of wage, w. In addition, V_u denotes the value of the unemployed worker, which is assumed to equal b. In the model, there will be flows between the pool of employed workers and the pool of unemployed worker. Some employed workers will be separated from their jobs and become unemployed, while some unemployed workers will receive job offers that are sufficiently attractive to accept.

1 (5 points) Let the reservation wage be defined as the wage such that the worker is indifferent between accepting and rejecting an offer. Please write down the equation that pins down the reservation wage in the model. Use the reservation wage to characterize the decision regarding accepting the job offer or not.

2 (5 points) Please use the reservation wage to compute the fraction of workers that change from unemployment to employment in each period.

3 (5 points) Suppose s denotes the separation rate, i.e., the fraction of workers who will become randomly separated from their jobs every period. Please write down the fraction of workers that change from employment to unemployment in each period.

4 (5 points) In the equilibrium, the number of worker that flows from employment to unemployment will equal the number of worker that flow from unemployment to employment. Use your answer in (2) and (3) to compute the equilibrium unemployment rate.

5 (5 points) Can you use the one-sided search model to analyze the effects of an increase in the unemployment insurance benefit, b, on the reservation wage and unemployment rate.

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3. Asset Pricing (25 points)

Consider an endowment economy with two periods, $t = 0, 1$. There are agents with mass one, and an agent endows $e_t = \bar{e}$ units of consumption goods at the beginning of period $t = 0, 1$. The consumption good is nondurable and will vanish after it is consumed. The agent's lifetime utility is

$$u(c_0) + \beta u(c_1),$$

where c_t is the consumption at period t, and $\beta \in (0, 1)$ is the time discount factor. There is a bond market that opens at period 0, and agents can purchase or issue one-period real bonds at the market. A one-period real bond issued at period 0 matures at period 1, and the issuer of the bond must pay 1 unit of consumption goods to the bond holder when the bond matures.

The bond market is competitive. Let b_0 denote an agent's net purchases of the bond, and let q_0 denote the equilibrium price of the one-period real bond in terms of period 0 consumption goods. Then the agent's period 0 and period 1 budget constraints are

$$\begin{aligned} c_0 + q_0 b_0 &= \bar{e}, \\ c_1 &= b_0 + \bar{e}. \end{aligned}$$

Note that the agents are the only participants in the bond market, but there is no government or any other people outside the economy engaging in the market.

(a) (10 points) Solve for the equilibrium price of the one-period real bond, q_0.

(b) (10 points) Now we change the model to make it three-period, so $t = 0, 1, 2$. An agent endows $e_t = \bar{e}$ units of consumption goods at the beginning of period $t = 0, 1, 2$. The agent's lifetime utility becomes

$$\sum_{t=0,1,2} \beta^t u(c_t).$$

Besides the one-period real bond, an agent can also purchase or issue a two-period real bond at the credit market at period 0. The two-period real bond is a zero-coupon bond. That means the issuer of a two-period bond pays 1 unit of consumption goods to the bond holder at period 2, but the issuer does not pay any interest at period 1 to the bond holder. Solve for the equilibrium price of the two-period real bond at period 0.

(c) (5 points) Now we introduce one more good, the houses, into the economy. As in question (b), an agent endows $e_t = \bar{e}$ units of consumption goods at the beginning of period $t = 0, 1, 2$. Moreover, an agent also owns $\bar{e}$ units of houses in the beginning of period 0. Houses are divisible, and houses provide living services to their holders: let h_t denote an agent's holding of houses at the beginning of each period, then holding h_t unit of houses generates $\alpha u(h_t)$ units of utility to the agent. Where $\alpha > 0$ represents the relative importance of housing utility to consumption utility. We assume that houses are perfectly durable, and that means houses do not vanish or depreciate over time or after they provide the living service.

Let an agent's consumption be c_t, and let her house holdings at the beginning of a period be h_t, then the agent's lifetime utility is

$$\sum_{t=0,1,2} \beta^t \left[u(c_t) + \alpha u(h_t)\right].$$

After agents receive the living service in each period, the housing market opens. The housing market allows agents to exchange between houses and consumption goods. Let p_t denote the price of houses in terms of consumption goods at period t. Solve for p_0 and p_1.

4. Heterogeneity (25 points)

Consider a one-period economy with competitive markets. There are mass one households and mass one firms. The firms use labor in units of working hours, n, to produce consumption goods, and the production function is:

$$f(n) = An,$$

where $A > 0$ captures the productivity of firms. The household's utility is

$$u(c, n) = \gamma c - \frac{1}{2}n^2,$$

where the c is the consumption, n is the working hour, and the parameter, $\gamma > 0$, captures the importance of consumption to households. A household's income comes from working, and the wage per hour is denoted by w. Thus, the household's budget constraint is

$$c = wn.$$

(a) (5 points) Solve for the equilibrium consumption, c, working hours, n, and wage rate, w.

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(b) (5 points) Now we extend the benchmark model to assume that households are heterogeneous. Consider that half of the households are type 1, and half of the households are type 2. Type 1 and type 2 value consumptions differently, and this is captured by their difference in the parameter γ. Type 1 households' utility is

$$u_1(c,n) = \gamma_1 c - \frac{1}{2}n^2;$$

and type 2 households' utility is

$$u_2(c,n) = \gamma_2 c - \frac{1}{2}n^2,$$

where $\gamma_1 > \gamma_2 > 0$. Let the ratio $\frac{c_1}{c_2}$ denote the consumption inequality. Solve for the ratio $\frac{c_1}{c_2}$ in the competitive equilibrium.

(c) (10 points) We further extend the model by introducing a government into the economy. The government charges a proportional tax on wage income. That is, suppose that a household's wage income is wn, then the government takes away twn from the household. The government transfers the tax revenue to all households equally in a lump sum manner. How does the proportional tax on wage influence the consumption equality $\frac{c_1}{c_2}$? Explain your answer.

(d) (5 points) Now we go back to the benchmark economy with homogenous households; that is, now the parameter γ is the same among households, and there is no government, tax, or transfer. However, we consider that firms are heterogeneous. That is, there are two types of firms. Type 1 firms' production function is

$$f_1(n) = A_1 n;$$

type 2 firms' production function is

$$f_2(n) = A_2 n,$$

and $A_1 > A_2$. Solve for the equilibrium wage rate and type 1 firms' and type 2 firms' labor hiring in units of working hours.