- 本試題共 7 大題, 合計 100 分。
- 請依題號依序作答。
- 請詳述理由或計算推導過程, 否則不予計分。

1. (15 %) Let $\{X_i\}_{i=1}^n \sim^{i.i.d.} N(\theta, \theta)$.

   (a) Find $Cov(X_i - \theta, (X_i - \theta)^2)$.

   (b) Find a pivotal quantity and use it to construct an *exact* 95% interval estimator of θ.

   (c) Consider the following estimator of θ:

   $$\hat{\theta}(c) = c\bar{X} + (1-c)\hat{\sigma}^2, \ \ \bar{X} = \frac{1}{n}\sum_i X_i, \ \ \hat{\sigma}^2 = \frac{1}{n}\sum_i (X_i - \bar{X}_n)^2, \ \ c \in [0,1]$$

   Find $Var(\hat{\theta}(c))$.

   Definition 1 *For an estimator T_n, if $\lim_{n\to\infty} k_n Var(T_n) = \tau^2 < \infty$, where $\{k_n\}$ is a sequence of constants (a normalizing constant), then τ^2 is called the limiting variance.*

   (d) Find the optimal choice of c that minimizes the limiting variance of $\hat{\theta}(c)$.

   State clearly what theorems/properties you use.

2. (10 %) Suppose that Y is discrete-valued, taking values only on the non-negative integers, and the conditional distribution of Y given $X = x$ is

   $$Y|X = x \sim \text{Poisson}(\beta x)$$

   (a) Can we estimate β by a linear regression model? Explain.

   (b) Does the model exhibit homoskedastic error structure? Explain.

3. (10 %) Suppose pseudo uniform random numbers on the interval $[0,1]$, $u_1, u_2, \ldots, u_n$, are generated by a specific algorithm.

   (a) Describe how to apply the inverse method to generate Logistic random variables X_i with pdf:

   $$f(x) = \frac{e^{-x}}{(1+e^{-x})^2}, \ \ x \in \mathbb{R}$$

   (b) Provide a theoretical justification for the inverse method.

4. (15 %) Consider the following growth regression model

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Regime_i + \beta_3 X_i \times Regime_i + \varepsilon_i$$

where $Y_i = \Delta\log(GDP_i)\times 100$ and $X_i = \Delta\log(export_i)\times 100$ are GDP growth and export growth, respectively. $Regime_1 = 1$ if the country adopts a fixed exchange rate system, and otherwise $Regime_1 = 0$.

(a) What is the impact of 1% increase in export growth on GDP growth for countries with a fixed exchange rate regime (i.e., fixers)?

Now suppose the empirical results are as follows:

$$\hat{Y}_i = \underset{(0.15)}{1.25} + \underset{(1.2)}{2.3}\ X_i - \underset{(0.22)}{0.56}\ Regime_i - \underset{(0.4)}{1.2}\ X_i \times Regime_i$$

where standard errors are in parentheses.

(b) Can you conclude (at 5% level of significance) that impacts of export growth on GDP growth vary by exchange rate regime?

(c) What is the predicted percentage difference in GDP growth of a fixer and a non-fixer having the same growth rate of export?

5. (20%) **True, false,** or **uncertain**? Evaluate the following statements with **brief** explanations.

(a) (5%) Multiplying the dependent variable by 100 and the explanatory variable by 1,000 makes the regression R^2 10 times larger.

(b) (5%) In a linear probability model, $Y_i = \beta_0 + \beta_1 X_i + u_i$, where Y_i is a binary variable and takes the value of **0** or **1**, the OLS estimate of β_1 is consistent and efficient.

(c) (5%) For the model

$$\begin{aligned} Y_i &= \beta_0 + \beta_1 X_i + u_i, \\ \mathrm{E}(u_i|X_i) &= 0. \end{aligned}$$

Let $\hat{\beta}_1$ be the OLS estimator of β_1 based on the available sample. Suppose that the i'th observation is included in the sample only if $Z_i = \gamma_0 + \gamma_1 W_i + v_i > 0$, and $\mathrm{Cov}(u_i, v_i) > 0$, then $\hat{\beta}_1$ is consistent.

(d) (5%) In the context of a controlled experiment, consider the simple linear regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, where Y_i is the outcome, X_i is the randomly assigned treatment level, and u_i contains all the additional determinants of the outcome. Then the OLS estimator of β_1 will be inconsistent since there are omitted variables present.

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6. (15%) Answer the following questions.

   (a) (4%) Consider the bivariate regression model, $Y_i = \beta_0 + \beta_1 X_i + u_i$, where $E(u|X) = 0$. Suppose that X_i is measured with error, and the measured X_i is $\tilde{X}_i = X_i + w_i$, where w_i is independent of X with variance σ_w^2. If we regress Y_i on $\tilde{X}_i$ using OLS, what is the probability limit of $\hat{\beta}_1$?

   (b) (7%) Consider the bivariate regression model with two-period panel data, $Y_{i,t} = \beta_0 + \beta_1 X_{i,t} + u_{i,t}$, where $X_{i,t}$ and $X_{i,t-1}$ are correlated with correlation coefficient $\rho_X > 0$. Suppose that $X_{i,t}$ is measured with error, and the measured $X_{i,t}$ is $\tilde{X}_{i,t} = X_{i,t} + w_{i,t}$, where $w_{i,t}$ is not autocorrelated and is independent of $X_{i,t}$, with variance σ_w^2. If we regress $(Y_{i,t} - Y_{i,t-1})$ on $(\tilde{X}_{i,t} - \tilde{X}_{i,t-1})$ using OLS, what is the probability limit of $\hat{\beta}_1^{FD}$ (first differenced $\hat{\beta}_1$)?

   (c) (4%) Is the bias of $\hat{\beta}_1^{FD}$ in (b) **greater** or **smaller** than the bias of $\hat{\beta}_1$ in (a)?

7. (15%) Consider the following simultaneous equations model:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + u_i, \quad (1)$$

$$X_i = \gamma_0 + \gamma_1 Y_i + \gamma_2 W_i + v_i, \quad (2)$$

where $E(u_i) = E(v_i) = 0$, $Cov(u_i, v_i) = Cov(Z_i, W_i) = 0$, and Z_i and W_i are exogenous.

   (a) (5%) Is the OLS estimate of β_1 consistent? Why?

   (b) (6%) Given the observations (X_i, Y_i, W_i, Z_i), $i = 1, \cdots, n$, describe the method and procedure to estimate β_1 **consistently**?

   (c) (4%) Is the proposed estimator in (b) consistent when $\gamma_2 = 0$? Why?

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