第一大題「單選題」與第二大題「複選題」考生應作答於「答案卡」(請勿作答於試卷之選擇題作答區)，未作答於答案卡者，該大題不予計分；非選擇題則請作答於「試卷」之非選擇題作答區。

j denotes $\sqrt{-1}$. $\mathcal{N}(\mu, \sigma^2)$ denotes Gaussian distribution with mean μ and variance σ^2. Unif($\mathcal{A}$) denotes uniform distribution over the set $\mathcal{A}$. $\overset{\text{i.i.d.}}{\sim}$ means " independent and identically distributed".

一、單選題：請作答於「答案卡」，依題號順序作答 (1., 2., ... , 21.)。

每題有三個選項。

I. (12%) Let

$$x[n] = \begin{cases} A, & n = 0 \\ B(1/2)^{|n|}, & 1 \le n \le 7 \end{cases}$$

be a periodic signal with period 8 and Fourier series coefficients a_k. Please determine whether each the following statements is (A) true, (B) false, or (C) uncertain (insufficient information to determine).

1. If $A = 2$, $B = 1$, then a_k is real. (3%)
2. If $A = 2$, $B = -1$, then a_k is odd. (3%)
3. If $A = 2$, $B = \mathrm{j}$, then a_k is real. (3%)
4. If $A = 2$, $B = -\mathrm{j}$, then a_k is even. (3%)

II. (10%) Let $x(t)$ be a signal with Nyquist rate of ω_0. Determine whether each of the following signals can undergo impulse-train sampling without aliasing given the stated sampling period ((A) true, (B) false or (C) uncertain).

5. $x(t) + 2$ with sampling period $T < 2\pi/\omega_0$. (2%)
6. $x^2(t) - x(t)$ with sampling period $T < \pi/\omega_0$. (2%)
7. $\mathrm{d}x(t)/\mathrm{d}t$ with sampling period $T < 2\pi/\omega_0$. (2%)
8. $x^*(t)$ with sampling period $T < 2\pi/\omega_0$. (2%)
9. $x(t)\sin\omega_0 t$ with sampling period $T < \pi/2\omega_0$. (2%)

III. (12%) Suppose we have the input signal

$$x[n] = \cos\left(\frac{\pi n}{6}\right) + 3\sin\left(\frac{\pi n}{3}\right)$$

to the LTI systems with the following impulse responses. Please determine whether each the following statements is (A) true, (B) false, or (C) uncertain (insufficient information to determine).

10. $y[n] = \sin(\frac{\pi n}{6})$ if $h[n] = \frac{\sin(\pi n/4)}{\pi n}$. (3%)
11. $y[n] = \cos(\frac{\pi n}{6}) + 3\sin(\frac{\pi n}{3})$ if $h[n] = \frac{\sin(\pi n/2)}{\pi n}$. (3%)
12. $y[n] = 2\cos(\frac{\pi n}{6}) + 3\sin(\frac{\pi n}{3})$ if $h[n] = \frac{\sin(\pi n/4)}{\pi n} + \frac{\sin(\pi n/2)}{\pi n}$. (3%)
13. $y[n] = 1/4\cos(\frac{\pi n}{6}) + \sin(\frac{\pi n}{3})$ if $h[n] = \frac{\sin(\pi n/4)\sin(\pi n/2)}{\pi^2 n^2}$. (3%)

IV. (16%) Given a stable and causal system with a real impulse response $h(t)$ and system function $H(s)$ satisfying the following properties:

- $H(s)$ is rational.
- One of its poles is at $-1+\mathrm{j}$.
- There is exactly one zero at infinity and no zero at the origin.

Please determine whether each the following statements is (A) true, (B) false, or (C) uncertain (insufficient information to determine).

14. $h(t)$ has finite duration. (2%)
15. ROC of $H(s)$ is $\mathrm{Re}\{s\} > -1$. (2%)
16. $\lim_{s\to\infty} H(s) = 3$. (2%)
17. $H(s) = H(-s)$. (2%)
18. $H(s)$ has more than 3 poles. (2%)
19. $H(\mathrm{j}\omega) = 0$ for at least one finite ω. (2%)
20. $h(t)e^{-2t}$ is absolutely integrable. (2%)
21. $\int_{-\infty}^{\infty} h(t)\,\mathrm{d}t = 2$. (2%)

二、複選題：請作答於「答案卡」，依題號順序作答 (22., 23.)。
每題有五個選項，至少有一選項為正確。每選對一選項得 +3 分，每選錯一選項得 −1 分 (倒扣)，整題不作答該題得 0 分。

22. (15%) A digital passband communication system employs QAM modulation with a constellation set to communicate through an additive white Gaussian noise (AWGN) channel. The noise power spectral density is $N_0/2$ and the average energy per symbol is E_s. Hence, the equivalent complex baseband channel model is

$$Y = X + Z,\ Z = Z_1 + \mathrm{j}Z_2,\ Z_1, Z_2 \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \tfrac{N_0}{2}).$$

The complex symbol X is chosen uniformly at random from the constellation set. Let $d_{\min}$ denote the minimum distance between constellation points of the constellation set. Let SNR denote E_s/N_0. Choose the correct statement(s) from the following.

(A) If the constellation set is 4PSK, then $d_{\min} = \sqrt{2E_s}$.

(B) If the constellation set is 4PSK, the optimal average symbol error probability $\mathsf{P_e}$ satisfies

$$\lim_{\mathsf{SNR}\to\infty} \left\{-\frac{1}{\mathsf{SNR}} \log_e(\mathsf{P_e})\right\} = \frac{1}{2}.$$

(C) If the constellation set is 4PAM, then $d_{\min} = \sqrt{5E_s}$.

(D) If the constellation set is 4PAM, the optimal average symbol error probability $\mathsf{P_e}$ satisfies

$$\lim_{\mathsf{SNR}\to\infty} \left\{-\frac{1}{\mathsf{SNR}} \log_e(\mathsf{P_e})\right\} = \frac{1}{5}.$$

(E) To achieve the same level of error probability, 4PAM requires less energy than 4PSK.

接次頁

23. (15%) Consider a wireless passband communication system. The center frequency is 2GHz and the total available bandwidth is 10MHz. QAM modulation is used, and the pulse shaper at the transmitter and the matched filter at the receiver are chosen to the root raised cosine pulse with roll-off factor β. Let $p(t)$ and $q(t)$ denote the pulse shaper and impulse response of the matched filter respectively. Let $g(t) = (p * q)(t)$, the convolution of $p(t)$ and $q(t)$. Choose the correct statement(s) from the following.

(A) $p(t) = q^*(-t)$.
(B) If $\beta = 0.25$, the number of real-valued symbols that can be sent per second is 8×10^5.
(C) If $\beta = 0$, the number of real-valued symbols that can be sent per second is 2×10^6.
(D) If $\beta = 0.25$, $\lim_{t\to\infty} tg(t) = 0$.
(E) If $\beta = 0$, $\lim_{t\to\infty} tg(t) = 0$.

三、非選擇題：請作答於「試卷」之非選擇題作答區。

24. (20%) Consider the binary detection problem of a real-valued symbol $U \sim \text{Unif}(\{a_0, a_1\})$ given three observations V_1, V_2, and V_3, where

$$V_1 = U + Z_1,\ V_2 = Z_1 + Z_2,\ V_3 = Z_2 + Z_3, \quad Z_1, Z_2, Z_3 \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2).$$

(i) Find the optimal detector and derive the optimal probability of error. Use the Q function (complementary CDF of a standard Gaussian) to express your answer. (8%)

(ii) Suppose there is an average energy constraint E_s, that is, $\mathsf{E}[|U|^2] \le E_s$. Find the best (a_0, a_1) such that the optimal probability of error is minimized. (6%)

(iii) In this detection problem, is (V_1, V_2) a sufficient statistic to detect U? If so, prove it. If not, find the additional energy needed to achieve the same probability of error as the optimal detector in Part (i) using the optimal constellation set in Part (ii). (6%)