第三大題選擇題(III. Multiple Choice Questions)考生應作答於「答案卡」

I. Calculation and proof (35%, 5% for each)

1. x is a random variable with positive variance. Let $u(x)=(x-b)^2$, b is any constant. Given existence of $E(u(x))$, find the b^* so that $E(u(x))$ is minimum.
2. Following previous question, denote $d=E(u(x))-u(E(x))$, please show that d is a function of b^*.
3. Denote new random variable $y=(x-b)^2$, and $f(x)=2x$, $0<x<1$. Please find p.d.f. of y.
4. Three events: A_1, A_2 and B are given. We know that A_1 and A_2 are disjointed events. $P(A_1)>0$ and $P(A_2)>0$. Let $P(B|A_1)=P(B|A_2)=0.2$. Please find $P(B|A_1\cup A_2)$.
5. Pikachu has 50% chance to tell a truth, and Jigglypuff has 90% of chance to tell a truth. Behaviors of Pikachu and Jigglypuff are independent. If a box contains 30 milk candies and 70 white chocolate candies. Now we randomly pick up one candy and Pikachu and Jigglypuff both share it, both of them claim that it is milk taste. Please find that the probability that candy is really a milk one.
6. Random variable x has the density function $f(x;\alpha)=\alpha\left(\frac{x}{2}\right)^{\alpha}$. Given five randomly sampled observations: 1, 1.1, 0.91, 2, 0.5, please find the maximum likelihood estimate for α.
7. A joint probability chart for two random variables X and Y is shown below.

	Y=1	Y=2	Y=3	Y=4
X=1	0.1	0	0	0
X=2	0.2	0.05	0.15	0
X=3	0.15	0.05	0.1	0.2

Please find
i) $E(X|Y=1)$. (2%)
ii) probability function of $E(X|Y)$. (3%)

II. True-False questions, please correct the statement with appropriate reasons if you respond 'false' (15%, 5% for each)

1. We model a random walk with drift model for y_t, the $E(y_t)$ does not depend on time t, while $Var(y_t)$ depends on time t.
2. Covariance stationary requires that mean and variance of the random variable are a constant.
3. Random sample $\{x_1, \ldots, x_n\}$ ~ Standard Cauchy distribution, and following central limited theorem, then $\bar{x}$~Normal distribution when n approaches infinite.

※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

III. Multiple Choice Questions (50%, 5% for each)

1. Consider the following joint probability distribution of CEO gender and firm profitability. Suppose $W=2+7X$ and $V=5+3Y$. What is the covariance between W and V?

	Low profit (X=0)	High profit (X=1)	Total
Female (Y=0)	0.2	0.1	0.3
Male (Y=1)	0.3	0.4	0.7
Total	0.5	0.5	1

(a) 0.25　(b) 0.21　(c) 0.5　(d) 1.05

2. $Y_i, i=1,\ldots,n.$ are i.i.d. Bernouli random variables with $p=0.3$. Let $\bar{Y}$ denote the sample mean. What

is the smallest sample size (n) that is needed to ensure that $Pr(\bar{Y} < 0.35)$ is at least 0.975? (Use the central limit theorem to compute an approximate answer.)

(a) 100　(b) 168　(c) 221　(d) 323

3. X and Z are two independently distributed standard normal random variables. $Y = X^2 + gZ$, g is a constant. Please calculate E(XY).

(a) 0　(b) 0.5　(c) 1　(d) 2.5

4. Suppose that a researcher, using data on corporate earnings and capital expenditure from 1000 companies, estimate the OLS regression:

$\widehat{Earnings} = 94 + 5.16 \times Capital\_Expenditure.$

All regression variables are measured in million dollars. The sample average capital expenditure across the 1000 firms is 60 million dollars. What is the sample average of the corporate earnings across the 1000 firms?

(a) 134 million dollars
(b) 286 million dollars
(c) 316 million dollars
(d) 404 million dollars

5. Consider a model relating a person's log wage to her education level and years of work experience. Suppose the true model is $\ln(Wage_i) = \beta_0 + \beta_1 Education_i + \beta_2 Experience_i + u_i$, and $Cov(Education_i, u_i) = 0$. Suppose that you estimate the following regression model instead: $\ln(Wage_i) = \gamma_0 + \gamma_1 Education_i + \epsilon_i$. Assume $\beta_2 > 0$ and $Cov(Education_i, Experience_i) < 0$. Which of the following statement is correct regarding the OLS (ordinary least squares) estimator $\hat{\gamma}_1$ in the regression model you have estimated?

(a) $\hat{\gamma}_1$ is a consistent estimator of β_1.
(b) $\hat{\gamma}_1 < \beta_1$ in a large sample.
(c) $E(\hat{\gamma}_1) = \beta_1$ in a small sample.
(d) $E(\hat{\gamma}_1) = e^{\beta_1}$ in a small sample.

Please use the following information to answer questions 6 and 7.

A random sample of 100 MBA students is selected from a population and these students' weight and height are recorded. A regression of weight (measured in pounds) on height (measured in inches) yields

$\widehat{Weight} = -50 + 3.21 \times Height, R^2 = 0.57, SER = 11,$

where SER refers to the standard error of the regression. Suppose now we change the unit of measurement from pounds to kilograms for weight and from inches to centimeters for height. (Note: 1 pound equals 0.454 kilogram and 1 inch equals 2.54 centimeters.)

6. What is the estimated regression intercept in the new regression model?

(a) -8.94　(b) -19.69　(c) -22.70　(d) -127.35

7. What is the estimated regression SER in the new regression model?

(a) 1.97　(b) 4.33　(c) 4.99　(d) 27.94

8. The interpretation of the slope coefficient in the model $\ln(Y_i) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$ is as follows:

(a) a 1% change in X_{1i} is associated with a β_1% change in Y_i, while keeping X_{2i} constant.
(b) a 1 unit change in X_i is associated with a change in Y_i of β_1%, while keeping X_{2i} constant.
(c) a 1% change in X_i is associated with a β_1 unit change in Y_i, while allowing X_{2i} to vary freely.
(d) a 1 unit change in X_i is associated with a change in Y_i of $\beta_1 \times 100\%$, while keeping X_{2i} constant.

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9. The true population model is $Y_i = \beta_0 + \beta_1 X_i + u_i$. Suppose that you estimate the following model by ordinary least squares: $\tilde{Y}_i = \gamma_0 + \gamma_1 X_i + \varepsilon_i$, where $\tilde{Y}_i$ is the estimate of Y_i, and $\tilde{Y}_i$ equals $Y_i + w_i$. Assume that w_i is not correlated with X. $\hat{\gamma}_1$ and $\hat{\beta}_1$ are the OLS estimators of γ_1 and β_1, respectively. Which of the following statements is correct?
(a) $E(\hat{\gamma}_1) > \beta_1$ and $Varinace(\hat{\gamma}_1|X) > Varinace(\hat{\beta}_1|X)$.
(b) $E(\hat{\gamma}_1) < \beta_1$ and $Varinace(\hat{\gamma}_1|X) < Varinace(\hat{\beta}_1|X)$.
(c) $E(\hat{\gamma}_1) = \beta_1$ and $Varinace(\hat{\gamma}_1|X) > Varinace(\hat{\beta}_1|X)$.
(d) $E(\hat{\gamma}_1) = \beta_1$ and $Varinace(\hat{\gamma}_1|X) < Varinace(\hat{\beta}_1|X)$.

10. An ordinary light bulb has a mean life of 835 hours and a standard deviation of 1000 hours. Jennifer is developing a new manufacturing process through which she hopes can improve the average life of a light bulb. She randomly selects 500 light bulbs produced by the new process. She says that she will believe that the new process works better than the old process if the sample mean life of the 500 light bulbs is greater than 885 hours; otherwise, she will conclude that the new light bulbs are no better than the old ones. Let μ denote the mean life (in hours) of the new light bulb. Consider the null and alternative hypotheses: H_0: $\mu = 835$ and H_1: $\mu =\!> 885$. What is the size of the plant manager's testing procedure? Please use Central Limit theorem to approximate the distribution of the t-statistics.
(a) 0　　(b) 0.065　　(c) 0.13　　(d) 0.26