※ 注意：請於試卷上「非選擇題作答區」內依序作答，並應註明作答之大題及其題號。

PART I: Financial Management

一、Multiple-Choice Question (4 points for each question)

1. The minimum variance portfolio is composed of two stocks: Nike and Adidas. The expected returns of Nike and Adidas are 12% and 14%, respectively. The variances of their returns are 6% and 8%, respectively. The covariance between their returns is 5%. Assume the risk-free rate is 4%. What are the minimum variance portfolio's weights on Nike (X_{Nike}) and Adidas (X_{Adidas})?
   (A) $X_{Nike} = 0.25$ and $X_{Adidas} = 0.75$　　(B) $X_{Nike} = 0.75$ and $X_{Adidas} = 0.25$
   (C) $X_{Nike} = 0.30$ and $X_{Adidas} = 0.70$　　(D) $X_{Nike} = 0.70$ and $X_{Adidas} = 0.30$
   (E) $X_{Nike} = 0.41$ and $X_{Adidas} = 0.59$

2. The tangency variance portfolio is composed of two stocks: Nike and Adidas. The expected returns of Nike and Adidas are 12% and 14%, respectively. The variances of their returns are 6% and 8%, respectively. The covariance between their returns is 5%. Assume the risk-free rate is 4%. What are the tangency variance portfolio's weights on Nike (X_{Nike}) and Adidas (X_{Adidas})?
   (A) $X_{Nike} = 0.25$ and $X_{Adidas} = 0.75$　　(B) $X_{Nike} = 0.75$ and $X_{Adidas} = 0.25$
   (C) $X_{Nike} = 0.30$ and $X_{Adidas} = 0.70$　　(D) $X_{Nike} = 0.70$ and $X_{Adidas} = 0.30$
   (E) $X_{Nike} = 0.41$ and $X_{Adidas} = 0.59$

3. The TSMC announces that it wins a major contract for its revolutionary computer chip from Apple. This very profitable contract will enable it to increase the growth rate of dividends from 6% to 7% without reducing the current dividend from the projected value of $5 per share. Assume that the constant-growth dividend discount model is valid and that the market capitalization rate for the TSMC is 15%. What is its original stock price ($P0$) before this announcement? What will be its new stock price ($P1$) after this announcement? What will happen to its future expected rate of return?
   (A) $P0 = \$55.56$; $P1 = \$55.56$; the future expected rate of return will be higher than before
   (B) $P0 = \$55.56$; $P1 = \$62.50$; the future expected rate of return will be higher than before
   (C) $P0 = \$55.56$; $P1 = \$66.88$; the future expected rate of return will be lower than before
   (D) $P0 = \$62.50$; $P1 = \$66.88$; the future expected rate of return will remain unchanged
   (E) $P0 = \$55.56$; $P1 = \$62.50$; the future expected rate of return will remain unchanged

見背面

4. NTU, Inc., has a debt to equity ratio of 4 to 1, using perpetual riskless debt. It has a beta of equity of 1.2. The risk-free rate is 5% and the market risk premium is 10%. The marginal corporate tax rate is 20%. Assume that the CAPM is true and the Modigliani-Miller assumptions hold. What is the company's weighted average cost of capital?
(A) 14.28%　(B) 16.12%　(C) 17.71%　(D) 18.49%　(E) 20.24%

5. All else constant, when the coupon rate of a bond is _____ the yield to maturity, it will sell at _____ .
(A) higher than; par　(B) higher than; a discount　(C) less than; a discount
(D) less than; a premium　(E) equal to; a premium

6. Generally, managers of corporations prefer internally generated cash to finance their capital expenditures because:
I) The costs of issuing new securities are high
II) The announcement of new equity issue is usually bad news for investors
III) They can avoid the discipline of the financial markets
IV) They can increase the liquidity of their stocks
(A) I only
(B) II only
(C) I and II only
(D) I, II, and III only
(E) All of the above

7. Which of the following statement(s) about information asymmetry between informed and uninformed traders is (are) correct if uninformed traders worry about the problem of adverse selection when they trade with informed traders?
I) Total trading volume is likely to increase before unscheduled corporate announcements (such as client and product announcements)
II) Total trading volume is likely to decrease before unscheduled corporate announcements (such as client and product announcements)
III) Total trading volume is likely to increase before scheduled corporate announcements (such as earnings announcements)
IV) Total trading volume is likely to decrease before scheduled corporate announcements (such as earnings announcements)
(A) I only
(B) I and III
(C) I and IV
(D) II and III
(E) II and IV

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8. The McGirwin has an average investment of $10,000 during the year. During the same time the firm has an after-tax earnings of $1,500. Its cost of capital is 8%. What are the net return on investment (ROI) and the economic value added (EVA) for the firm?
(A) ROI = 7%; EVA = $800　　(B) ROI = 7%; EVA = $700
(C) ROI = 8%; EVA = $800　　(D) ROI = 15%; EVA = $700
(E) ROI = 15%; EVA = $800

9. An investor needs cash to pay some bills. He is willing to use his dividend income to pay the bills, but he will not sell any stock to do so. He is engaging in _____.
(A) forecast errors
(B) representativeness
(C) overconfidence
(D) mental accounting
(E) conservatism

10. A project has an expected risky cash flow of $500 in year 1. The risk-free rate is 5%, the market rate of return is 15%, and the project's beta is 1.2. Calculate the certainty equivalent cash flow for year 1.
(A) $427.35　(B) $448.72　(C) $456.34　(D) $476.19　(E) $482.63

二、Proof Question (10 points)

Consider the following simple efficient markets model in which the real price P_t of a stock at the beginning of the time period t is given by:

$$P_t = \sum_{k=0}^{\infty} \rho^{k+1} E_t[D_{t+k}] \qquad 0 < \rho = \frac{1}{1+r} < 1$$

where D_t is the real dividend paid at the end of time t, E_t is the expectations operator conditional on information available at time t, ρ is the constant real discount factor, and r is the constant real interest rate. The information set at time t (Ω_t) includes P_t and D_t and their lagged values, and will generally include other variables as well. Define the one-period holding return as $HR_t \equiv \frac{P_{t+1} - P_t + D_t}{P_t}$. Prove that $E_t[HR_t] = r$.

見背面

第二部份財金數學

A. 多重選擇題 (每題選項至少有一個是正確的，每題須全對才給分，但答錯並不倒扣。)

以下各題所談到的矩陣，皆為有限維的**實數矩陣**，。以下符號各指的是: rank(·)為秩、$tr(\cdot)$為跡(trace)、RS(·)為列空間(row space)、CS(·)為行空間(column space)、det(·)為行列式值(determinant)、$M_{3\times3}(R)$為收集三階實數矩陣的向量空間、mod 模除(modulo)、$max(x,y)$為取 x 與 y 中的最大值、dim(·)為維度(dimension)。

1. **(5%)** 設 A 為一矩陣，其尺寸大小為 10×3，其中 $\text{rank(A)}=3$。今定義 $H=A(A^TA)^{-1}A^T$，以及I_{10}為十階的單位矩陣，請問下列敘述哪些是正確的?
   (A) H 是正定矩陣。
   (B) $(I_{10}+H)$ 的反矩陣一定是存在的。
   (C) $rank(I_{10}-H)=7$。
   (D) $tr(H)=3$。
   (E) 對於任意$b\in CS(A)$，此線性方程組$Hx=b$必定有解。

2. **(5%)** 設 A 為一個 10 階的對稱且正定的矩陣，請問下列敘述哪些是正確的?
   (A) A 一定可對角化(diagonalizable)。
   (B) A 一定可以做喬列斯基分解(Cholesky decomposition)。
   (C) A 的反矩陣，一定也是對稱且正定的。
   (D) 考慮 A 矩陣中任意兩個線性獨立的特徵向量，這兩個特徵向量必內積等於零。
   (E) 考慮此矩陣A^2，其所有的特徵值一定都是正值。

3. **(5%)** 設 A 為一矩陣，其尺寸大小為$m\times n$，其中$m>1$ 以及 $n>1$。請問下列敘述哪些是正確的?
   (A) 若$RS(A)=CS(A)$，則$A^T=A$。
   (B) 若$A^T=-A$，則$RS(A)=CS(A)$。
   (C) 若$\det(A)=1$，則$RS(A)=CS(A)$。
   (D) $\text{rank}(AA^T)=\text{rank}(A^TA)$。
   (E) AA^T的特徵值會跟A^TA的特徵值完全相同。

4. **(5%)** 設A、B、C為三個n階方陣，以及I_n為 n 階的單位矩陣，請問下列敘述哪些是正確的?
   (A) $\det(-A)=-\det(A)$ 。
   (B) 若 A 與 B 的反矩陣皆存在，則 $\det\big((A+B)(A-B)\big)=\det(A^2)-\det(B^2)$。
   (C) 若 $A=CBC^{-1}$ ，則 $\det(A-nI_n)=\det(B-nI_n)$。
   (D) 若 $\text{rank}(A^2)<\text{rank}(A)$，則$\dim\big(N(A^2)\big)>\dim(N(A))$
   (E) 不存在A與B使得$AB-BA=I_n$。

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5. **(5%)** 請問下列哪些代數系統(G,⊕)滿足代數群(group)的結構?

(A) G = {所有的非負整數}，運算 ⊕：$a \oplus b = max(a,b)$。

(B) G = $\{x \in$ 複數集 $|x^4 = 1\}$，運算 ⊕：$a \oplus b = ab$。

(C) G = {1,2,3,4}，運算 ⊕：$a \oplus b = ab \ (mod\ 5)$。

(D) G = $\{A \in M_{3\times3}(R) | A^T = A\}$，運算 ⊕：$A \oplus B = AB$。

(E) G = $\{P \in M_{3\times3}(R) | P^2 = P\}$，運算 ⊕：$P \oplus Q = PQ$。

備註: mod 5 指的是除以 5 的餘數，例如 7 (mod 5) = 2 ; 3 (mod 5) = 3。

B. 計算題 (共兩大題，每題都必須附上計算過程)

一、(13%) 假設市場上有兩股票 X,Y，其今日之股價分別為 18 和 16，且兩股票均不發放股利。假設一年後，X,Y 股票價格會受到三種狀態的影響，其分別對應的股價如下表:

股票 / 狀態	X	Y
State 1	25	15
State 2	15	20
State 3	20	17.5

(1) (5%) 考慮一或有請求權(contingent claim) $C = [C_1 \quad C_2 \quad C_3]^T$。若此 C 可被市場上這兩支股票 X,Y 所複製，則C_1, C_2, C_3必須滿足一個線性的限制式，請求此限制式。

(2) (8%) 在無套利假設下，請問此市場一年的無風險報酬率是多少?(請以百分率呈現)

二、(12%) 求解下列這個微分方程系統(請務必清楚表示求解過程!)

$$\begin{cases} \dfrac{dx_1}{dt} = 2x_1 - 3x_2 \\ \dfrac{dx_2}{dt} = 3x_1 + 2x_2 \end{cases}$$

試題隨卷繳回