1. **[General Equilibrium]** (50 points)

Consider the following two period general equilibrium model with a representative firm and representative household. The household owns the firm and receives profits from the firm as their income. The firm owns capital.

1. The two period firm's problem:

In the first period t, the firm starts from K_t units of capital to produce $Y_t = A_t K_t^{1/2} = 4K_t^{1/2}$, where A_t is productivity (assumes $A_t = 4$ now). The output and capital are the same goods sold at prices p_t and p_{t+1} in period *t* and *t*+1. Y_t can be used as consumption or investment. In the second period, *t+1*, the firm uses $K_{t+1} = I_t + (1-d)K_t$ and produces $Y_{t+1} = 4K_{t+1}^{1/2}$. I_t is the firm's investment and *d* is the capital depreciation rate. The nominal interest rate between *t* and *t+1* is i_t. Note that the real interest rate is $(1+\mathrm{r}) = \frac{p_t}{p_{t+1}}(1+i_t)$

In the first period after production, the firm sells output to the household and determines investment that it carries to the next period. The profit at t is $\pi_t = p_t Y_t - p_t I_t$. At the next period, the firm produce and sell all its output and capital to the household as consumption. So the profit at t+1 is $\pi_{t+1} = p_{t+1}Y_{t+1} + p_{t+1}(1-d)K_{t+1}$.

a. By investing, the firm reduces its profit at *t* but increases K_{t+1}. What is the formula of the gross return (payoff/cost) from capital at t? (5 points)

b. What is the user cost of capital here? (5 points)

c. Assume $p_t = p_{t+1} = 10,\ i_t = 1/11,\ d = 1/11,\ K_t = 100$. What is the desired optimal capital at *t+1*, K_{t+1} ? (5 points)

2. The household's problem

The household receives firms' profits as the income in each period. The household maximizes his lifetime utility as:

$$\max_{C_t,\, C_{t+1}} U(x_1, x_2) = ln(C_t) + 0.8\ln(C_{t+1})$$

Subject to his lifetime budget constraint,

$$C_t + \frac{C_{t+1}}{1+r} = \frac{\pi_t}{p_t} + \frac{\pi_{t+1}/p_{t+1}}{1+r}.$$

Where *r* is the real interest rate, and $\frac{\pi_t}{p_t}$ is real income at *t*.

a. What is the household's Euler equation ?(5 points)
b. What is the marginal propensity to consume today? (5 points)

3. General Equilibrium

a. Write down the market clearing conditions.(5 points)
b. Now, assume $K_t = 4, d = 1,\ p_t = p_{t+1} = 2, A_t = A_{t+1} = 4$, the interest rate is unknown and needs to be determined in equilibrium. Calculate the general equilibrium values of $C_t,\ C_{t+1}, I_t, K_{t+1}, r$. List clearly five equations associated with these 5 endogenous variables. (10 points)
c. Following from b., Suppose now there is a productivity shock so that A_t increases from 4 to, but A_{t+1} remains the same. What is the new equilibrium interest rate? Does it increase or decrease? (10 points)

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2. **[Labor and Government Policy]** (25 points) Consider an one-period, production economy. There are infinite households and firms in the economy, and the mass of households and firms are both normalized to one.

The household's utility function is

$$u(c, l) = \ln c + \ln l,$$

where c is consumption, and l is leisure. A household can work to obtain wage income, and wage income is equal to the wage rate w times the working hours n. A household's endowment of time is normalized to 1, and thus its time constraint is

$$n + l = 1.$$

Households also own shares of firms, and the dividend payment they receive from firms is π.

There is a government. The government makes an expenditure $g > 0$. The size of the expenditure is exogenously determined and does not bring any utility to the households. The expenditure is funded by a lump sum tax, τ charged from households. The government's budget constraint is

$$g = \tau,$$

and the household's budget constraint is

$$c = wn + \pi - \tau.$$

A firm takes working hours n as inputs, and its production function is

$$F(n) = An,$$

where A is a technology parameter. A firm's profit comes from the revenue of selling output, $F(n)$, minus the wage payment, wn. A firm maximizes its profit, and the profit is transferred to households as dividend payments, π.

(a) (5 points) Write down the social planner's problem. Solve for the optimal working hours, n, and consumption, c, in the problem.

(b) (10 points) Now we consider the competitive equilibrium.

i. Compare the competitive equilibrium working hours, n, and consumption, c, with those in the social planner's problem.

ii. In an economy with a higher government's expenditure, g, will the working hours, n, consumption c, and wage rate, w, be higher, lower, or the same in the competitive equilibrium?

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(c) (10 points) Now suppose that the government makes no expenditure, so $g = 0$. However, the government provides a proportional subsidy to households based on their wage income. That is, let the subsidy rate be $t \geq 0$, then if a household's wage income is wn, the government pays twn to the households. Moreover, the subsidy is funded by a lump sum tax charged from households, τ. Thus, let $\bar{n}$ denote the average labor supply among households, and let t denote the government's transfer to a household, then

$$\tau = tw\bar{n}.$$

The households' budget constraint becomes

$$c = (1+t)wn + \pi - \tau.$$

i. Are the **consumption** and **working hours** in the competitive equilibrium higher in an economy with a proportional subsidy ($t > 0$) or without a proportional subsidy ($t = 0$)? Or the proportional subsidy t has no impact on the households' decision?

ii. Is **social welfare** in the competitive equilibrium higher in an economy with a proportional subsidy ($t > 0$) or without a proportional subsidy ($t = 0$)? Or the proportional subsidy t has no impact on the social welfare? **Explain your answer.**

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3. [**Economic Growth**] (25 points) Consider a Solow growth model with no population growth and no technological progress. Time is discrete: $t = 0, 1, 2, \ldots$. The aggregate production function is

$$Y = F(K, N)$$

Where Y, K, and N are the aggregate output, aggregate capital, and aggregate labor. The households' behavior on consumption and investment is as follows: there is an exogenous saving rate $s \in (0, 1)$, and a household uses s proportion of it output to invest and consume the rest, so

$$\begin{aligned} I_t &= sY_t, \\ C_t &= (1-s)Y_t. \end{aligned}$$

Moreover, the accumulation of the aggregate capital follows

$$K_{t+1} = (1-\delta)K_t + I_t.$$

Where δ is the depreciation rate of capital. Let $k_t = \frac{K_t}{N_t}$ denote the capital per capita.

(a) Let $F(K, N) = AK^{\alpha}N^{1-\alpha}$, where $0 < \alpha < 1$.

i. (5 points) Use the accumulation rule of the aggregate capital to derive the law of motion for capital per capita k_t.

ii. (5 points) Solve for the steady state levels of capital per capita and consumption per capita.

(b) Now let $F(K, N) = AK_t^{\alpha}N^{1-\alpha+\gamma}$, where $0 < \alpha < 1$ and $\gamma > 0$. Suppose that the economy was initially at a steady state, and the steady state capital per capita is equal to $k^* > 0$. An unexpected earthquake occurs at $t = 5$, and 50 percent of the capital and population are destroyed.

i. (5 points) Compare the steady state levels of **capital per capita** before the earthquake and after the earthquake.

ii. (10 points) Identify from Figure 1 the closest dynamic paths of **capital per capita** and **output per capita** in the economy.

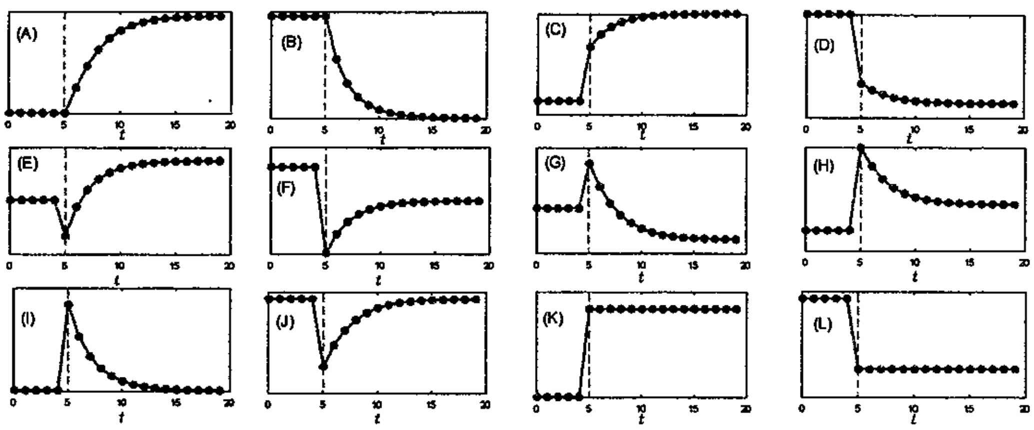

Figure 1: Dynamic paths

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