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國立臺灣大學108學年度碩士班招生考試試題

科目：常微分方程

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1. (25 pts) Let

$$\mathbf{A} = \begin{pmatrix} 1 & -8 & 8 \\ 0 & 4 & 1 \\ 0 & -6 & 9 \end{pmatrix}.$$

(a) Find $e^{\mathbf{A}t}$.

(b) Solve $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t) + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{x}(0) = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$.

2. (25 pts) Solve the following equations.

(a) $\frac{dy}{dx} = \frac{y^2}{xy + x^2}$.

(b) $\frac{dy}{dx} - \frac{b(x)}{x}y = x^2, x > 0; y(1) = 3,$

where

$$b(x) = \begin{cases} 1 & \text{for } 0 < x \leq 2, \\ 2 & \text{for } x > 2. \end{cases}$$

(Assume $y(x)$ is continuous and piecewise differentiable.)

3. (25 pts) Let

$$V_1 = \{y(t) \mid y(t) \text{ is a solution of } y'' - ty' + 2y = 0, 0 \leq t \leq 1\},$$

$$V_2 = \{z(t) \mid z(t) \text{ is a solution of } 3z''' + (t^2 - 2)z'' + tz' - 4z = 0, 0 \leq t \leq 1\},$$

$$V_3 = \{y(t) + z(t) \mid y(t) \in V_1, z(t) \in V_2\}.$$

Find the dimensions of the vector spaces V_1, V_2 and V_3 . Explain your reasons.

4. (25 pts)

(a) Assume $\beta > 0$. Solve the equation $y'' + y = 2\cos(\beta t)$, $y(0) = 3$, $y'(0) = 0$.

(b) Find the limit $\lim_{\beta \rightarrow \infty} y(t)$.

(c) Let $f(t)$ be a continuous function satisfying $f(t + 2\pi) = f(t)$ and $f(-t) = f(t)$. Assume that $z(t)$ is the solution of $z'' + z = f(\beta t)$, $z(0) = 0$, $z'(0) = 1$. Find $\lim_{\beta \rightarrow \infty} z(t)$.

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