1. **(12 points)** Please solve the following questions:
(a) (6 points) Bowl A contains three red and two white chips, and bowl B contains four red and three white chips. A chip is drawn at random from bowl A and transferred to bowl B. Compute the probability of then drawing a red chip from bowl B.
(b) (6 points) An urn contains two red balls and four white balls. Sample successively five times at random and with replacement, so that the trials are independent. Compute the probability of each of the two sequences *WWRWR* and *RWWWR*.

2. **(13 points)** Let X_1, X_2, X_3 denote a random sample of size $n = 3$ from a distribution with the geometric pmf

$$f(x) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1},\ x = 1,2,3,.......$$

(a) (3 points) Compute $P(X_1 = 1, X_2 = 3, X_3 = 3)$

(b) (5 points) Determine $P(X_1 + X_2 + X_3 = 6)$

(c) (5 points) If Y equals the maximum of X_1, X_2, X_3, find $P(Y \le 2) = P(X_1 \le 2)P(X_2 \le 2)P(X_3 \le 2)$.

3. **(6 points)** Let μ and σ^2 denote the mean and variance of the random variable X. Determine

(a) (3 points) $E\left[\frac{(X-\mu)}{\sigma}\right]$

(b) (3 points) $E\left[\left(\frac{X-\mu}{\sigma}\right)^2\right]$

4. **(9 points)** If the moment-generating function of X is $M(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t}$, find the mean, variance, and pmf of X.

5. **(10 points)** Let $f(x;\theta) = \theta x^{\theta-1},\ 0 < x < 1,\ \theta \in \Omega = \{\theta : 0 < \theta < \infty\}$. Let $X_1, X_2, ... X_n$ denote a random sample of size n from this distribution.
(a) (4 points) Sketch the pdf of X for **(i)** $\theta = 1/2$, **(ii)** $\theta = 1$,
(b) (6 points) Show that $\hat{\theta} = \frac{-n}{\ln\left(\prod_{i=1}^{n} X_i\right)}$ is the maximum likelihood estimator of θ.

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6. **(50 points)** Consider the standard simple regression model: $y = \beta_0 + \beta_1 x + u$ under the Gauss-Markov Assumptions. The OLS estimators for β_1 and β_0 are

$\hat{\beta}_1 = [\Sigma_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})]/\mathrm{SST_x}$ and $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$, where $\mathrm{SST_x} = \Sigma_{i=1}^{n}(x_i - \bar{x})^2$.

The residuals are defined as: $\hat{u}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$, for $i = 1,2,\ldots,n$.

From normal equation we know $\Sigma_{i=1}^{n} x_i \hat{u}_i = 0$.

(a) (**5** points) Suppose that $\mathrm{E(u)} = \alpha \neq 0$. Show that the model can always rewritten with the same slope, but a new intercept and error, where the new error has a zero expected value.

(b) (**5** points) What are the **five** Gauss-Markov assumptions for the simple regression?

(c) (**10** points) Show that $\hat{\beta}_1 = \hat{\rho}_{xy} \cdot \hat{\sigma}_y / \hat{\sigma}_x$, where $\hat{\rho}_{xy}$ is the sample correlation and $\hat{\sigma}_x$ and $\hat{\sigma}_y$ denote the sample standard deviations.

(d) (**10** points) Show that R^2 is equal to the square of the sample correlation coefficient between y and $\hat{y}$.

(e) (**20** points) Let $c_1 > 0$ and $c_2 > 0$ be constants. Regress $c_1 y_i$ on $c_2 x_i$. Find the new intercept and slope estimators in terms of $\hat{\beta}_1$ and $\hat{\beta}_0$. (10 points) Again, regress (y_i/c_1) on $(x_i - c_2)$. What will be the new intercept and slope estimators? (10 points)

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