國立臺灣大學 108 學年度碩士班招生考試試題

1. **(20 points)** Consider the following two linear programming (LP) models.

LP1	**LP2**
$\max z = c_1x_1 + c_2x_2$	$\max z = 10c_1x_1 + 10c_2x_2$
subject to	subject to
$a_{11}x_1 + a_{12}x_2 \le b_1$	$10a_{11}x_1 + 10a_{12}x_2 \le b_1$
$a_{21}x_1 + a_{22}x_2 \le b_2$	$10a_{21}x_1 + 10a_{22}x_2 \le b_2$
$x_1, x_2 \ge 0$	$x_1, x_2 \ge 0$

Assume that the basic variables of the optimal solution for **both** LPs are x_1 and x_2. The optimal solution of LP1 is x_1=30, x_2=300, z=330, and the shadow prices of constraints 1 and 2 in LP1 are the same and equal to 100/3.

a. (10 points) Find the optimal solution and the objective value to LP2.
b. (5 points) Formulate the dual problem for LP2.
c. (5 points) Find the optimal solution and the objective value to the dual problem of LP2.

2. **(20 points)** Consider a supply chain that consists of two manufacturers (say, M1 and M2) and two retailers (R1 and R2). These two manufacturers sell one product to the retailers, and M1 and M2 can provide 2 and 3 units of the product, respectively. R1 and R2 will purchase 1 and 4 units from both manufacturers, and the shipping cost per unit from M1 to R1 is 3, from M1 to R2 is 1, from M2 to R1 is 2, and from M2 to R2 is 3.

a. (10 points) Formulate the problem as an **assignment problem** to minimize the total shipping cost.
b. (10 points) Solve the problem by using the Hungarian method.

3. **(10 points)** (2 points for each) Consider the following statements about any pure integer programming (IP) problem (in maximization form) and its LP relaxation. Answer True or False for each statement.

a. The number of the feasible solutions is finite for the pure IP problem with a bounded feasible region.
b. IP problems are generally easier to solve than their LP relaxation.
c. The feasible region for the LP relaxation is a subset of the feasible region for the IP problem.
d. If an optimal solution for the LP relaxation is an integer solution, the optimal value of the objective function will be the same for both problems.
e. If a noninteger optimal solution is found for the LP relaxation, the nearest integer solution (rounding each variable either up or down) is optimal to the IP problem.

4. **(20 points)** ***IIE*** is a travel agency located in Taipei. ***IIE*** arranges 1-week tours for international tourists. To fulfill customer demands, ***IIE*** needs 8, 5, 8, and 9 tour buses over the next 4 weeks, respectively. The needs for tour buses can be met through renting additional buses or returning unneeded buses. Both renting and returning activities incur costs. Renting a bus in any week will incur a fixed cost of \$600 plus \$330 per bus per week, and returning a bus in any week will incur a fixed cost of \$600. ***IIE*** may choose to keep excess buses in any week by simply continue paying the rent of \$330 per week.

The goal is to minimize overall costs while fulfilling all the needs for tour buses.

a. (10 points) Find the optimal numbers of buses renting and returning in each of the next four weeks.
b. (10 points) Find the total cost under the optimal renting and returning policy.

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5. **(15 points)** ***ORMS*** operates an ice-cream truck and sells ice-cream in one of the following four locations: Zoo, Museum, Aquarium, and Beach. On the average, the daily revenue of the ice-cream truck is:

- \$1,700 at the Zoo,
- \$1,200 at the Museum,
- \$1,100 at the Aquarium, and
- \$1,300 at the Beach.

The location of the ice-cream truck is predictable, but the location may change every day. Every morning, there is a 45% probability that ***ORMS*** will move to a different location and a 55% chance that ***ORMS*** will sell ice-cream at the same location as the day before. When the ice-cream truck moves, there are equal probabilities of switching to one of the other three locations.

a. (5 points) ***ORMS*** is selling ice-cream at the Zoo this Monday. Find the probability that ***ORMS*** will sell ice-cream at the Museum this Friday.

b. (10 points) Find the long-run average revenue of the ice-cream truck per day.

6. **(15 points)** *NTU* produces and sells two products, Product A and Product B.

- If *NTU* sells q_1 units of Product A, the price its customers are willing to pay for Product A is $210 - 12q_1$ dollars.
- If *NTU* sells q_2 units of Product B, the price its customers are willing to pay for Product B is $450 - 45q_2$ dollars.
- The production cost is: $300 + 45(q_1 + q_2)$.

a. (15 points) The goal is to maximize the total profit. Find the optimal total profit and the optimal quantities q_1 and q_2.

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