1. Find the general solutions of the differential equations:

   I. $y'' - \frac{1+x}{x}y' + \frac{1}{x}y = 0$ (5%)

   II. $y'' + xy' + y = 0$ (5%)

   III. $y'' + \frac{1+x}{x}y' + \frac{x-1}{x^2}y = 0$ (5%)

2. Find the Taylor series about 0 of the solution to the following initial-value problem: (15%)

$$y'' - 2xy' + 8y = 0$$
$$y(0) = 4, \qquad y'(0) = 0$$

3. If $u = 1 + \frac{x^3}{3!} + \frac{x^6}{6!} + \cdots, v = \frac{x}{1!} + \frac{x^4}{4!} + \frac{x^7}{7!} + \cdots, w = \frac{x^2}{2!} + \frac{x^5}{5!} + \frac{x^8}{8!} + \cdots$, prove that $u^3 + v^3 + w^3 - 3uvw = 1$ (10%)

4. A (real) square matrix U is called unitary if $U^T = U^{-1}$. Show that the absolute value of any eigenvalue of U is unity. (10%)

5. The equations of an ellipse in parametric form are:
   $x = a\cos\theta \quad y = b\sin\theta$
   Find the equation of the tangent line to the curve at the point (x = 1, y = 2) in terms of the constant a and b. (10%)

6. If $u = e^{3y}\cos 2x$, what is du/dt if both x and y are functions of t? (10%)

7. Given the two differential equations:

$$\frac{dy}{dt} - \lambda z = 0$$
$$\frac{dz}{dt} + \lambda y = 0$$

   Solve for y and z if, at t = 0, y = 0 and z = 0 (λ is a constant) (10%)

8. An open-topped hemispherical bowl, with an inside radius of 10 ft, is filled with water. The water flows out of a hole in the bottom. The rate of flow 5 cfm at the instant the water level gas dropped 4 ft. What is the rate of change of the height of the water surface at this instant in feet per minute? (10%)

9. A bin with a square base, straight sides, and no top is to be constructed from 432 ft2 of lumber. Find the dimensions of the bin such that the capacity of the bin is a maximum. What is the maximum volume? (10%)

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