1. Consider a linear system whose augmented matrix is of the form (20%)

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array}\right]$$

(a) For what values of a and b will the system have infinitely many solutions?
(b) For what values of a and b will the system be inconsistent?

2. Let $A=\begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$ where all four blocks are $n\times n$ matrices. (20%)

(a) If A_{11} and A_{22} are nonsingular, show that A must also be nonsingular and that A^{-1} must be of the form

$$\begin{bmatrix} A_{11}^{-1} & C \\ 0 & A_{22}^{-1} \end{bmatrix}$$

(b) Determine C.

3. Apply the Gram-Schmidt process to $A = [\vec{a} \quad \vec{b} \quad \vec{c}]$, where $\vec{a}=[0 \quad 0 \quad 1]^T$,

$\vec{b}=[0 \quad 1 \quad 1]^T$, $\vec{c}=[1 \quad 1 \quad 1]^T$, and write the result in the form $A = QR$. (15%)

4. What are the eigenvalues λ, eigenfrequencies ω, and the general solution of the equation? (15%)

$$\frac{d^2u}{dt^2}=\begin{bmatrix} -5 & 4 \\ 4 & -5 \end{bmatrix}u.$$

5. True or false, with a reason: (20%)
   (a) An invertible matrix can't be similar to a singular matrix.
   (b) A symmetric matrix can't be similar to a nonsymmetric matrix.
   (c) A can't be similar to $-A$ unless $A = 0$.
   (d) $A - I$ can't be similar to $A + I$.
6. Identify the curve and sketch the graph. $x^2 + xy + y^2 = 6$. (10%)

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