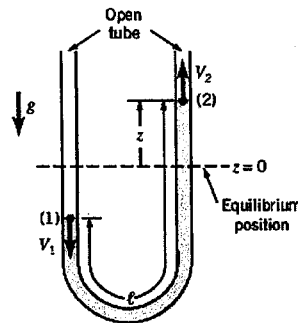
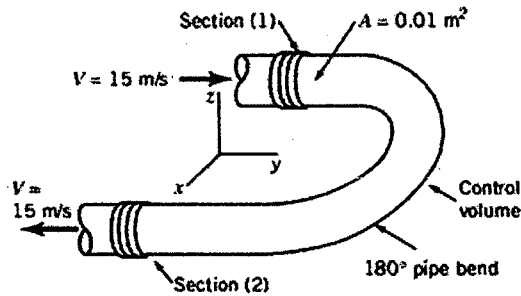


1. (25%) An incompressible, inviscid liquid is placed in a vertical, constant diameter U-tube. When released from the non-equilibrium position, the liquid column will oscillate at a specific frequency. Determine this frequency. Hint: the unsteady form of the Bernoulli equation is (where  $V$  is the local velocity along the streamline)

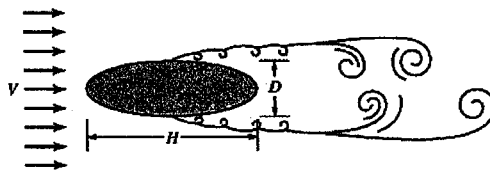
$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = \rho \int_{s_1}^{s_2} \frac{\partial V}{\partial t} ds + p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$



2. (25%) Water flows through a horizontal, 180-degree pipe bend. The cross-sectional area is  $0.01m^2$  through the bend. The flow velocity in the bend is axial and  $15 m/s$ . The absolute pressure at the entrance and exit of the bend are 207 kPa and 165 kPa, respectively. Calculate the horizontal (x and y) components of the force required to hold the bend in place.



3. (25%) A long object (prototype) has an elliptical cross section. When a steady wind (air) blows past this object, vortices develop regularly at some frequency. For the prototype,  $D = 0.1m$ ,  $H = 0.3m$ , and the wind velocity is  $V = 50km/hour$ . The density of air is  $\rho_{air} = 1.23 kg/m^3$  and the dynamic viscosity of air is  $\mu_{air} = 1.79 \times 10^{-5} kg/m \cdot s$ . The frequency at which vortices develop is determined through a small-scale model tested in a water tunnel. For the model,  $D_m = 20mm$  and the density of water is  $\rho_{water} = 998 kg/m^3$  and the dynamic viscosity of water is  $\mu_{water} = 1 \times 10^{-3} kg/m \cdot s$ . Determine the model dimension  $H_m$  and the velocity  $V_m$  at which the test should be performed? If the shedding frequency for the model is  $\omega_m = 49.9Hz$ , what is the corresponding frequency for the prototype?



4. (25%) Two immiscible, incompressible, viscous fluids having the same densities but different viscosities are contained between two infinite, horizontal, parallel plates as shown. The bottom plate is fixed and the upper plate moves at the constant velocity  $U$ . Determine the velocity at the interface. Express your answer in terms of  $U$ ,  $\mu_1$ ,  $\mu_2$ . The motion of the fluid is caused by the upper plate and no pressure gradient exists in the x direction. The velocity and shear are continuous across the interface and the flow is laminar.

