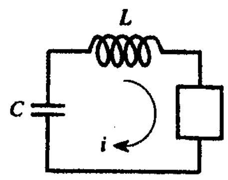
1. (30%) The box in the circuit shown in the figure represents an "active element" such as a semiconductor or vacuum tube, the voltage drop across which is a known function $f(i)$ of the current i. Thus, Kirchhoff's voltage law gives $L\frac{di}{dt}+f(i)+\frac{1}{C}\int i\,dt=0$.



(a) If f is of the form $f(i)=ai^3-bi$, show that one obtains

$$Li''+(3ai^2-b)i'+\frac{1}{C}i=0. \qquad (1)$$

(b) Show that by a suitable scaling of both the independent and dependent variables one can obtain from (1) the van der Pol equation

$$I''-e(1-I^2)I'+I=0,$$

where primes denote differentiation with respect to the new time variable τ, where $t=\alpha\tau$ and $i=\beta I$. That is, find α, β, and e in terms of L, C, a, and b.

2. (20%) Evaluate the Jacobian,

$$f(u,\upsilon,w)=uw^3, \quad g(u,\upsilon,w)=2\upsilon-w,$$
$$h(u,\upsilon,w)=e^{uv}; \quad \frac{\partial(f,g,h)}{\partial(u,\upsilon,w)}$$

3. (20%) Derive the Fourier integral representations of the following functions. At which points, if any, does the Fourier integral fail to converge to $f(x)$?

$$f(x)=\begin{cases} e^{-x}, & x\ge 0 \\ 0, & x<0 \end{cases}$$

4. (30%) Verify the divergence theorem by working out $\int_V \nabla\cdot\mathbf{v}\,dV$ and $\int_S \hat{\mathbf{n}}\cdot\mathbf{v}\,dA$ and showing that the results are equal.

$$\mathbf{v}=(3x^2-2yz)\hat{\mathbf{j}},$$

ς: the pentahedron with vertices at $(0,0,0)$, $(2,0,0)$, $(0,0,3)$, $(2,0,3)$, $(0,4,3)$, $(2,4,3)$

試題隨卷繳回