1. (25%) (a)若考慮一黏度計(Viscometer)利用兩同心旋轉圓柱進行液體黏度的量測，該同心圓柱的長度夠長，邊壁效應可以忽略。該兩同心圓柱各以不同轉速旋轉，可以利用在單一圓柱在切線方向所承受的每單位長度的扭矩(T_z)、圓柱的半徑(r_o,r_i)、與圓柱的角速度(ω_o,ω_i)來決定待測液體的黏滯係數(Viscosity)，下標 o 代表外圓柱，i 代表內圓柱。而兩同心圓柱間的液體切線方向的速度(v)可以用下列公式表示

$$\frac{d(v/r)}{dr}=\frac{\tau_{r\theta}}{\mu r}=\frac{A}{\mu r^3},$$

其中A是一常數，$\tau_{r\theta}$是剪應力，請推導出黏滯係數與內圓柱切線方向所承受的扭矩(T_z)、圓柱的半徑(r_o,r_i)、與圓柱的角速度(ω_o,ω_i)之間的關係(20%) (b)若外圓柱固定不動，(r_o-r_i=g << r_i)，請將所推導的方程式簡化成黏滯係數與內圓柱切線方向所承受每單位長度的扭矩(T_z)、兩同心圓柱間距(g)、內圓柱的半徑(r_i)、與內圓柱的角速度(ω_i)之間的關係(5%)。

2. (25%)在紊流流場內，若要描述速度分佈，會採用與層流邊界層不同的參數為座標來描述，稱為 Law of the Wall，請說明 Law of the Wall 座標中橫軸參數與與縱軸參數的定義(8%)，其中的摩擦速度(friction velocity, u^*)如何定義也請說明(5%)。請在壁座標(Wall Coordinate)分別繪出沒有壓力梯度與有很強的逆向壓力梯度(Very Strong Adverse Pressure Gradient)下，在紊流流體經過一平板的速度分佈 (12%)。

3. (25%) Consider the components of the velocity field of a flow are

$$u(x,y,z,t)=\frac{Ax}{\left(x^2+y^2+z^2\right)^{3/2}},\quad v(x,y,z,t)=\frac{Ay}{\left(x^2+y^2+z^2\right)^{3/2}},\quad w(x,y,z,t)=\frac{Az}{\left(x^2+y^2+z^2\right)^{3/2}},\quad x,y,z\neq 0,$$

where u,v,w are the components in $x,y,z-$ directions, respectively, and A is a constant. Assume that the gravity is negligible and the density of the fluid is uniform, i.e., $\rho=\rho(t)$. The dynamic viscosity of the fluid μ is a constant.
(a) Is this flow incompressible? (5%) (b) Is this flow irrotational? (5%) (c) Compute the rate of strain tensor of this flow. (5%) (d) What are the principal strain-rates. (5%) (e) If the pressure at infinity is P_∞, what is the pressure field of this flow? (5%)

4. (25%) When wind blows on a skyscraper like Taipei 101, the resulting forces cause the building to vibrate. This flow-induced vibration of skyscraper can be approximated by the flow-structure interaction of wind and a vertical cylindrical cantilever beam. Let's say the vertical the beam has a height H, a diameter D, a modulus of elasticity E, and a moment of inertia I about the ground. The density of the air is ρ and the dynamic viscosity of the air is μ. The wind speed is U_∞ and the vibrating frequency is f. (a) Construct a dimension table with information regards the relevant dimensional parameters and important primary dimensions. (5%) (b) Determine all the relevant dimensionless groups. (15%) (c) Construct a functional relation between the dimensionless vibrating frequency and the other dimensionless groups and simplify the functional relation as much as possible. (5%)

試題隨卷繳回