

You can answer your questions in English or Chinese.

1. (20 points) Consider the incompressible, two-dimensional flow of a non-viscous fluid between the boundaries shown in the following figure. The velocity potential for this flow field is

$$\phi = x^2 - y^2$$

- (a) Draw the flow net for this flow.  
 (b) Determine the corresponding stream function  
 (c) What is the relationship between the discharge  $q$  passing between the walls and the coordinates,  $x_i, y_i$  of any point on the curved wall? Neglect body forces.  
 (d) If the pressure on the wall ( $x=2\text{m}, 0$ ) is 75 kPa, what is the pressure on the other point of the wall ( $x=0, y=1\text{m}$ ), assuming there is no elevation difference between these two points ( $x$ - $y$  plane is horizontal)?

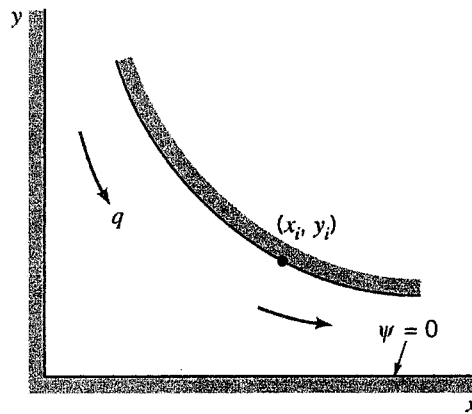


Figure 1

2. (20 points) Figure 2 shows fluid with density  $\rho$  flowing through a two-dimensional conduit, whose width and length are  $b$  and  $h$ , respectively. At the entrance of the conduit, the velocity is  $u_0$  and is uniformly distributed. The pressure at the entrance is  $p_A$ . At the exit of the conduit, the velocity profile is a parabola, given by

$$u = U \left[ 1 - \left( \frac{2y}{b} \right)^2 \right]$$

where  $U$  is the maximum value of the velocity at the exit cross section and  $y = 0$  represents the centerline. The pressure at the exit cross section is given by

$$p_B = p_A - 2.25\rho \frac{U^2}{2}$$

- (a) Determine the relationship between  $u_0$  and  $U$ .  
 (b) Determine the force applied per unit width of the conduit.

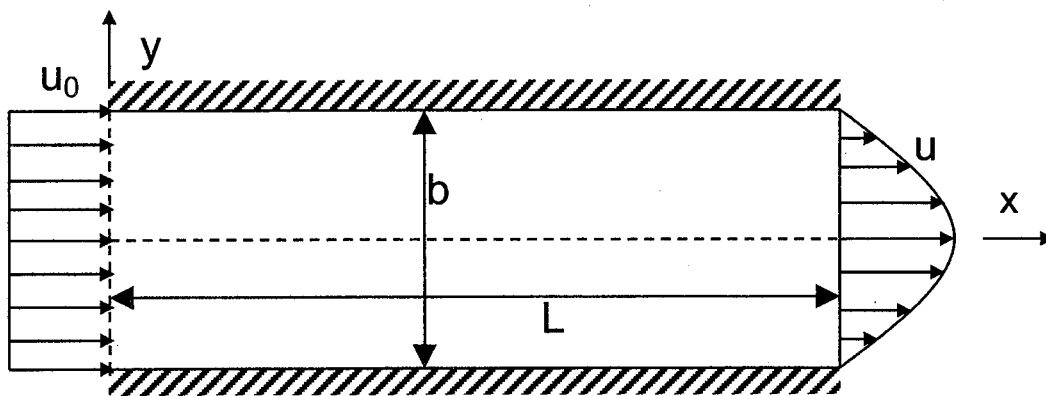


Figure 2

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3. (20 points) The experimental measurements show that the pressure drop  $\Delta P$  along the axis of a cylindrical pipe of uniform cross section depends on the following variables:

Pipe diameter  $D$  of the pipe

Axial length of the pipe  $l$

The fluid density  $\rho$

Dynamic viscosity  $\mu$

Gravitational constant  $g$

The composition of the inner surface denoted by an absolute roughness height  $\epsilon$ . Average fluid velocity  $V$  over the pipe cross section. Obtain a set of dimensionless parameters using the PI theorem. Choose  $\mu$ ,  $D$  and  $V$  as the repeating variables. Also explain the physical meaning of each dimensionless parameter applied.

4. (20 points)

(a) Describe what is the difference between the Navier-Stokes equations and the Euler equations.

(b) Navier-Stokes equations and the mass balance equation (4 differential equations) can be solved for 4 unknowns for fluids as a function of space and time. Can you describe those four unknowns?

(c) What is a "fully developed flow"?

5. (20 points) The components of a velocity field are given by  $u = \frac{10y}{(x^2+y^2)^{1/2}}$ , and  $v = -\frac{10x}{(x^2+y^2)^{1/2}}$ .

(a) Describe the steadiness and dimensionality (1D, 2D or 3D?) of the flow.

(b) Determine the local and convective acceleration, respectively.

(c) Determine the streamline and streakline for the above flow field.

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