

1. Consider an economy with two time periods (labelled 0 and 1) in which a typical consumer has preferences given by: $\log(c_0) + \beta \log(c_1)$, where c_t is consumption in period t . Each consumer is endowed with k_0 units of capital at the beginning of period 0 and with one unit of time in each period.

In each period, there is a price-taking, profit-maximizing firm that produces goods using capital and labor. These goods can be either consumed or saved in the form of capital that can be used in production in the next period. Let the firm's production function be: $y = zk^\alpha n^{1-\alpha}$, where k is the amount of capital rented by the firm and n is the amount of labor rented by the firm in a given period.

Because leisure is not valued (leisure does not appear in the utility function), each consumer supplies labor inelastically, i.e., he supplies one unit of labor in each time period. The only interesting decision that a consumer makes, then, is how much to save in period 0. Let k_1 be the amount of capital that a typical consumer saves in period 0. Then each consumer faces a pair of budget constraints:

$$c_0 = r_0 k_0 + w_0 - k_1$$

and

$$c_1 = r_1 k_1 + w_1,$$

where r_t is the rental price of capital in period t (expressed in terms of period- t consumption goods) and w_t is the wage rate in period t (again expressed in terms of period- t consumption goods). Each consumer takes these prices as given when deciding how much to save. Because period 1 is the last period of his life, each consumer consumes all of his resources in period 1.

In equilibrium, the markets for goods, labor, and capital must clear in both time periods.

- (a) Find an explicit expression (in terms of primitives) for the competitive equilibrium capital stock in period 1. Use your answer to determine the equilibrium rate of return on savings between periods 0 and 1. In addition, determine the equilibrium allocation of consumption across the two time periods. How do changes in z affect the equilibrium allocation? Explain. (10 points)
- (b) Formulate a social planning problem for this economy and show that the allocation chosen by the social planner is identical to the competitive equilibrium allocation that you determined in part (a). (5 points)
2. Consider the following model with a pollution externality. The representative consumer has preferences given by the utility function
- $$U(c, l, x) = u(c, l) - v(x)$$
- where $u(\cdot)$ is strictly concave and increasing in both arguments, and $v(\cdot)$ is increasing and strictly convex. Here, c denotes consumption, l is leisure, and x is the quantity of pollution produced by the representative firm. The representative firm's technology for producing consumption goods is given by
- $$y = zn,$$
- where y is output of the consumption good, n is the labor input, and $z > 0$. When the firm produces consumption goods, it also produces pollution, which is equal to αn where $\alpha > 0$.
- (a) Suppose that there are markets only in labor and consumption goods. Let w denote the wage rate. Show that the competitive equilibrium is not Pareto optimal. Is there too much or too little pollution in equilibrium? Explain your results. (10 points)
- (b) Now suppose that the government levies a proportional tax, t , on output, and distributes the proceeds of the tax in a lump-sum fashion to the consumer. That is, if τ is the lump-sum transfer to the consumer, then the government must satisfy its budget constraint $ty = \tau$. Show that there is a tax rate, t , such that the competitive equilibrium is Pareto optimal, and explain your results. (10 points)

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- (c) Alternatively, suppose that, in addition to markets in labor and consumption goods, there is a market in pollution rights. That is, the consumer can sell the rights to the firm to pollute, at a price of p units of consumption goods per unit of pollution produced. Show that the competitive equilibrium is Pareto optimal, and explain why. (10 points)
3. China has a fixed exchange rate against the U.S. dollar. Suppose the U.S. economy experiences a severe recession. Would China be better off with a fixed exchange rate, or should it allow its exchange rate to float? Does your answer depend on *why* the U.S. economy experiences a recession? Use the Mundell-Fleming model to illustrate your answer. (5 points)
4. Consider a neo-classical growth model with two sectors. One sector produces the market produced consumption good, c_{mt} , using a constant-returns-to-scale technology, $F^1(k_{1t}, h_{1t})$. The second sector produces the investment good, i_t , using the constant-returns-to-scale technology, $F^2(k_{2t}, h_{2t})$. The economy is populated by a large number of identical households with preferences given by, $\sum_{t=0}^{\infty} \beta^t U(c_{mt}, l_t)$, where $0 < \beta < 1$. Households are endowed with one unit of time that can be allocated to working in the two sectors and to leisure, l_t . The stock of capital in each sector depreciates at a common rate, δ . It takes one period to produce productive capital. Thus, the total stock of capital at period t , k_t , evolves according to $k_{t+1} = (1 - \delta)k_t + i_t$.
- (a) Assume that capital can be freely moved from one sector to another. Formulate the social planner's problem for this economy. (5 points)
- (b) Now assume that capital can NOT be moved between sectors. Formulate the social planner's problem for this economy. (5 points)
- (c) Follow the assumption that capital can be freely moved from one sector to another. However, now we assume that it takes TWO periods to produce productive capital. In other words, investment in period t will become productive capital in period $t+2$. Formulate the social planner's problem for this economy. (10 points)
- (d) Using the assumption in (a), write down the household's maximization problem. Write down the firms' problems. Provide the market clearing conditions in a competitive equilibrium for this economy. (10 points)

In addition to market produced good, now we consider home produced consumption good, c_{ht} . The home good is produced using "experience" (a_t) and time (h_{3t}). In particular, $c_{ht} = F^3(a_t, h_{3t})$, where $a_{t+1} = (1 - \rho)a_t + h_{3t}$. Preferences are now given by $\sum_{t=0}^{\infty} \beta^t U(c_{mt}, c_{ht}, l_t)$. Therefore, now there are two consumption goods and three sectors (one home production and two market productions) in the economy. In the following questions, we focus on the assumptions that capital is freely moved between sectors and it takes one period to produce productive capital.

- (e) Formulate the social planner's problem for this economy. (10 points)
- (f) Write down the household's maximization problem. Write down the firms' problems. Provide the market clearing conditions in a competitive equilibrium for this economy. (10 points)

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