- 本試題共 7 大題, 合計 100 分。
- 請依題號依序作答。
- 請詳述理由或計算推導過程, 否則不予計分。

1. (10%) Let $\varepsilon_t \sim^{i.i.d.} (0,1)$, and $e_t = \varepsilon_t \varepsilon_{t-1}$.

   (a) (5%) Is e_t a martingale difference sequence with respect to $\mathcal{F}_t = \{\varepsilon_t\}$?

   (b) (5%) Is e_t an i.i.d. sequence?

2. (20%) Let
$$\{X_i\}_{i=1}^n \sim^{i.i.d.} \text{Bernoulli}(p)$$
The odds ratio is defined as $\theta = \frac{p}{1-p}$.

   (a) (5%) Find the maximum likelihood estimator of θ.

   (b) (5%) Construct a 95% confidence interval of θ.

   (c) (5%) Suppose that you have obtained $\{x_1, x_2, \ldots, x_n\}$ as the realizations of a random sample. Describe how to construct a 95% percentile bootstrap confidence interval of θ.

   (d) (5%) Suppose that the realizations of the random sample have 80 successes and 20 failures. Use the above data to test $H_0 : \theta = 3$ vs. $H_1 : \theta > 3$ with significance level $\alpha = 0.01$.

3. (5%) Let $(X, Y, Z) \sim^{i.i.d.} N(0,1)$. Find the distribution of
$$W = \frac{X + YZ}{\sqrt{1 + Z^2}}$$

4. (15%) Consider the linear model
$$Y = \alpha + \beta X + \varepsilon$$
where ε is an error term such that $Cov(X, \varepsilon) \neq 0$.

   (a) (5%) Is $\alpha + \beta X$ the best predictor of Y given X?

   (b) (5%) Let Z be a random variable satisfying $E(\varepsilon|Z) = 0$. What is the best predictor of Y given Z?

   (c) (5%) Now suppose that X and Z are bivariate normal random variables:
$$\begin{pmatrix} X \\ Z \end{pmatrix} \overset{d}{=} N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$$
   where ρ is a known constant. Use this information to determine α and β. What happens if $\rho = 0$?

[Some useful $N(0,1)$ probabilities]

$$P(N(0,1) \leq 2.33) = 0.99, \quad P(N(0,1) \leq 1.96) = 0.975$$
$$P(N(0,1) \leq 1.64) = 0.95, \quad P(N(0,1) \leq 1.28) = 0.90$$

見背面

5. (20%) **True, false,** or **uncertain,** and **Why**? Evaluate the following statements with **brief** explanations.

   (a) (5%) Under the principle of presumption of innocence, letting a guilty person go free is a Type III error.

   (b) (5%) According to the Gauss-Markov theorem, the OLS estimate is unbiased when the error terms are heteroscedastic.

   (c) (5%) Suppose that variable Z is a determinant of the dependent variable Y, omitted variable bias will definitely occur if Z is not included in the regression of Y on X.

   (d) (5%) The probit model is always better than the linear probability model when the dependent variable is *binary* even if the error term is heteroscedastic.

6. (10%) The parameter β is defined in the model

$$Y_i = \beta X_i^* + u_i$$

where u_i is independent of X_i^*, $E(u_i) = 0$, $E(u_i^2) = \sigma^2$, the observables are (Y_i, X_i) where

$$X_i = X_i^* v_i$$

and $v_i > 0$ is random measurement error. Assume that v_i is independent of X_i^* and u_i. Also assume that X_i and X_i^* are non-negative and real-valued. Consider the least-squares estimator $\hat{\beta}$ for β.

   (a) (5%) Find the probability limit of $\hat{\beta}$, expressed in terms of β and moments of (X_i, v_i, u_i).

   (b) (5%) Find a condition under which $\hat{\beta}$ is consistent for β?

7. (20%) Consider the following regression model

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

and let Z be a *binary* instrumental variable (IV) for X.

   (a) (4%) What are the conditions for Z to be a valid instrument for X?

   (b) (5%) What is the IV estimator of β_1 ?

   (c) (5%) Is the IV estimator of β_1 consistent? Explain.

   (d) (6%) Show that the IV estimator of β_1 can be simplified as a function of $\bar{Y}_0$, $\bar{X}_0$, $\bar{Y}_1$ and $\bar{X}_1$, where $\bar{Y}_0$ and $\bar{X}_0$ are the simple averages of Y_i and X_i over the part of the sample with $Z_i = 0$, and $\bar{Y}_1$ and $\bar{X}_1$ are the simple averages of Y_i and X_i over the part of the sample with $Z_i = 1$.