1. (15%) Let $T_{(1)},\ldots,T_{(n)}$ be the order statistics of a random sample $T_1,\ldots,T_n$ from an exponential distribution with rate λ_0. Derive the joint distribution of $U_1,\ldots,U_n$, where $U_i=(n-i+1)(T_{(i)}-T_{(i-1)})$, $i=1,\ldots,n$, with $T_{(0)}=0$.

2. (10%) (10%) Let $X_1,\ldots,X_n$ be a random sample from a population with the density function $f(x|\theta_0) = 0.5e^{-|x-\theta_0|}I_{\{(-\infty,\infty)\}}(x)$. Find the maximum likelihood estimator of θ_0 and derive its sampling distribution.

3. (15%) Let $X_1,\ldots,X_{n+1}$ be a random sample from $Bernoulli(\pi_0)$ and $h(\pi_0)=P(\sum_{i=1}^{n}X_i > X_{n+1}|\pi_0)$. Find the uniformly minimum variance unbiased estimator of $h(\pi_0)$.

4. (5%) (10%) State and show the weak law of large numbers.

5. (15%) Suppose that $\widehat{\theta}$ is an estimator of θ with $E[\widehat{\theta}]=\theta+b_1/n+b_2/n^2+\ldots$. The sample is further split into p groups of size m with $n=mp$. Let $\widehat{\theta}_j$ be computed from the $m(p-1)$ observations left after the jth group has been deleted, $V_j=p\widehat{\theta}-(1-p)\widehat{\theta}_j$, $j=1,\cdots,p$, and $\widehat{\theta}_J=\sum_{j=1}^{p}V_j/p$. Show that the bias of $\widehat{\theta}_J$ is of the order n^{-2}.

6. Let $X_1,\ldots,X_n$ be a random sample from a population with the density function $f(x|\theta) = \theta e^{-\theta x}I_{(0,\infty)}(x)$.

(6a) (10%) Find a size α, $0<\alpha<1$, likelihood ratio test of $H_0:\theta=\theta_0$ versus $H_A:\theta\neq\theta_0$.

(6b) (10%) Find a valid p-value for the above hypotheses.

試題隨卷繳回